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DocMus Applied Study Scientific / Artistic Research
First Doctoral Presentation, 19.10.24, Organo



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Chords, melodies: a look at harmony by numbers



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Chords, melodies: a look at harmony by numbers
Tuning Bach — an experimental recomposition



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Chords, melodies: a look at harmony by numbers
Sara Cubarsi and Xenia Gogu, violins



Streams barely in winter (2019) 1. Cold 2. Sun 3. Stones
three beginnings for Walter played before Bach for two violins

Sei Bach-Intonazioni per Violino Solo (2000–17) Ia IIa IIIa
version in just intonation for violin solo with violin bordun counterpoint

Chords, melodies: a look at harmony by numbers
Sara Cubarsi and Xenia Gogu, violins



Harmony by numbers

- harmony ?



Harmony by numbers

- harmony ?
- numbers ?



Harmony by numbers | “*armonia*” = fitting together

- harmony
- numbers ?



Harmony by numbers | “*armonia*” = fitting together

- harmony = relations between pitches other than higher / lower (Tenney)
- numbers ?



Harmony by numbers | *“overtones”* = aliquot divisions

- harmony = relations between pitches other than higher / lower
- numbers



Harmony by numbers | *“overtones”* = aliquot divisions

- harmony = relations between pitches other than higher / lower
- numbers = harmonic partial row (Aristotle, Mersenne, Sauveur, Helmholtz, et al.)



Harmony by numbers | *“overtones”* = aliquot divisions

- harmony = relations between pitches other than higher / lower
- numbers = harmonic partial row; ratios of partials = rational intervals (Partch et al.)



Harmony by numbers = relations of harmonic partials

- harmony = relations between pitches other than higher / lower
- numbers = harmonic partial row; ratios of partials = rational intervals



Harmony by numbers = rational intonation (JI)

- harmony = relations between pitches other than higher / lower
- numbers = harmonic partial row; ratios of partials = rational intervals



Harmony by numbers: research questions

- What is rational intonation or JI?



Harmony by numbers: research questions

- What is rational intonation or JI? *a practice of intonation by rational intervals*



Harmony by numbers: research questions

- What is rational intonation or JI? *a practice of intonation by rational intervals*
- What does it sound like?



Harmony by numbers: research questions

- What is rational intonation or JI? *a practice of intonation by rational intervals*
- What does it sound like? *intervals, chords, timbres, melodies ...*



Harmony by numbers: research questions

- What is rational intonation or JI? *a practice of intonation by rational intervals*
- What does it sound like? *intervals, chords, timbres, melodies ... tools?*



Harmony by numbers: research questions

- What is rational intonation or JI? *a practice of intonation by rational intervals*
- What does it sound like? *intervals, chords, timbres, melodies ... tools? ... notation*



Harmony by numbers: research questions

- What is rational intonation or JI? *a practice of intonation by rational intervals*
- What does it sound like? *intervals, chords, timbres, melodies ... tools? ... notation*
- Repertoire?



Harmony by numbers: research questions

- What is rational intonation or JI? *a practice of intonation by rational intervals*
- What does it sound like? *intervals, chords, timbres, melodies ... tools? ... notation*
- Repertoire: how does one make decisions about tuning an existing piece in JI?



Harmony by numbers: research questions

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making choices that prioritise tuneable, consonant intervals sometimes requires microtonal pitch variations based on context (i.e., microtonal modulations)



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- How can a performer learn to hear and play such tunings accurately?



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making choices that prioritise tuneable, consonant intervals sometimes requires microtonal pitch variations based on context (i.e., microtonal modulations)
- How can a performer learn to hear and play such tunings accurately?
- How does JI relate to historically aware performance practices of earlier music?



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pythagorean, meantone, well-tempered tuning systems



Harmony by numbers: research questions

- What is rational intonation or JI? *a practice of intonation by rational intervals*
- What does it sound like? *intervals, chords, timbres, melodies ... tools? ... notation*
- Repertoire: how does one make decisions about tuning an existing piece in JI? *making choices that prioritise tuneable, consonant intervals sometimes requires microtonal pitch variations based on context (i.e., microtonal modulations)*
- How can a performer learn to hear and play such tunings accurately?
- How does JI relate to historically aware performance practices of earlier music? *pythagorean, meantone, well-tempered tuning systems*
- What new forms of music are emerging from JI practices? How can knowledge about intonation shape the creation of new music?



Case Study 1: Tuning Bach

Tuning Bach features an experimental recomposition of J.S. Bach's three solo violin sonatas: an adaptation for two violins in rational intonation by Marc Sabat.



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- basic questions: is it possible to tune Bach's solo violin works by strictly following the logic of rational intonation, using intervals tuneable by ear?



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- basic questions: is it possible to tune Bach's solo violin works by strictly following the logic of rational intonation, using intervals tuneable by ear? how might it sound?



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In the mid 2010's, I began composing a second violin part, with the intention of providing sustaining tones that allow JI intervals to be tuned more precisely by ear; gradually this evolved into a contrapuntal conversation with Bach's music.



Case Study 1: Streams barely in winter

Tuning Bach features an experimental recomposition of J.S. Bach's three solo violin sonatas, an adaptation for two violins in rational intonation by Marc Sabat.

In 2019, after making the Bach intonations, three short preludes were composed, one for each sonata. These miniatures for two violins take emblematic JI elements of the respective Bach sonatas as points of departure, introducing the sounds of various kinds of JI intervals.



Case Study 1: recording and live concert

Tuning Bach features an experimental recomposition of J.S. Bach's three solo violin sonatas, an adaptation for two violins in rational intonation by Marc Sabat.

In 2019, after making the Bach intonations, three short preludes were composed, one for each sonata. These miniatures for two violins take emblematic JI elements of the respective Bach sonatas as points of departure, introducing the sounds of various kinds of JI intervals.

Both the preludes and the adaptations were recorded in a studio setting and edited for upcoming release. This lecture-concert, presented as part of my doctoral research, marks the first live performance of the complete project.



Harmony by numbers: a brief tour of JI Fundamentals

- the harmonic partial row



Harmony by numbers: a brief tour of JI Fundamentals

- the harmonic partial row
- HEJI Notation



Harmony by numbers: a brief tour of JI Fundamentals

- the harmonic partial row
- HEJI Notation
- JI intervals and chords



Harmony by numbers: a brief tour of JI Fundamentals

- the harmonic partial row
- HEJI Notation
- JI intervals and chords
- listening techniques for tuning JI sounds



Harmony by numbers: harmonic partials

form a sequence of frequencies which are whole-number multiples of a generating pitch, called the fundamental.



Harmony by numbers: harmonic partials

form a sequence of frequencies which are whole-number multiples of a generating pitch, called the fundamental. Each new harmonic partial fits into the vibrating period of the fundamental, creating a fused harmonic sound. As partials are added, the colour or *timbre* of the sound is enriched or brightened. As lower partials disappear, the sense of fundamental (or periodicity pitch) lingers, creating a *virtual fundamental* enhanced by *combination* or *summation and difference tones*. These *psychoacoustic* phenomena form the basis for how combinations of pitches, even the slightly detuned ones found in the equal tempered system, form harmonies (intervals, chords, aggregates, scales and modes). The concept of *just* or *rational intonation* (JI) is based on the untempered tunings found in the harmonic partial row. *Extended JI* refers to harmonies including the microtonal higher prime partials, 7° , 11° , 13° , etc.



Harmony by numbers: notating JI (HEJI)

Every partial that is a multiple of only 2's and 3's is part of a sequence of perfect fifths and fourths, commonly called "Pythagorean" tuning (with a history dating back to Ancient Mesopotamia). It is written with "normal" flats, naturals, or sharps.



Harmony by numbers: odd partials = new pitch-classes

Every partial that is a multiple of only 2's and 3's is part of a sequence of perfect fifths and fourths, commonly called "Pythagorean" tuning (with a history dating back to Ancient Mesopotamia). It is written with "normal" flats, naturals, or sharps.

Multiplying partials by 2 transposes them up an octave. Therefore, *even-numbered* partials are octave transpositions of pitches occurring previously, while *odd-numbered* partials represent the first (and lowest) occurrences of new pitch classes.



Harmony by numbers: prime partials = new accidentals

Every partial that is a multiple of only 2's and 3's is part of a sequence of perfect fifths and fourths, commonly called "Pythagorean" tuning (with a history dating back to Ancient Mesopotamia). It is written with "normal" flats, naturals, or sharps.

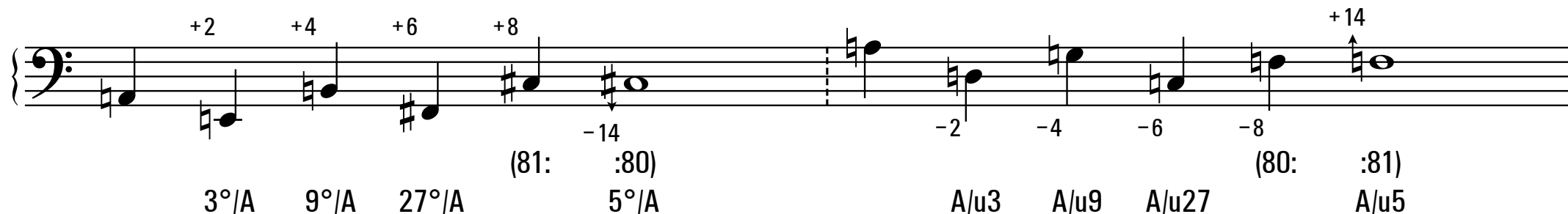
Odd partials that are *prime* numbers always need an additional *modifying* accidental to notate microtonal deviation from a nearby "normal" note. The inverted version of an accidental shows the same deviation occurring *below* a reference pitch.

Harmony by numbers: the Syntonic Comma

Every partial that is a multiple of only 2's and 3's is part of a sequence of perfect fifths and fourths, commonly called "Pythagorean" tuning (with a history dating back to Ancient Mesopotamia). It is written with "normal" flats, naturals, or sharps.

Odd partials that are *prime* numbers always need an additional *modifying* accidental to notate microtonal deviation from a nearby "normal" note. The inverted version of an accidental shows the same deviation occurring *below* a reference pitch.

Notation of 5° partial and its inversion u5 as deviations of a Syntonic Comma from the nearby Pythagorean notes





The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

revised by Marc Sabat & Thomas Nicholson

*in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz*

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above ♮A

Standard otonal notation above ♮A. The notation shows a series of notes on a staff, with microtonal accidentals (numbers) above them. The notes are labeled with intervals and ratios.

Interval	Ratio	Microtonal Accidental
partial interval alteration		+2
M3	(81:80)	-14
m7	(64:63)	-31
P4	(32:33)	+4
M6	(27:26)	+51
aug8	(2187:2176)	+5
m3	(512:513)	+4
aug4	(729:736)	+51
		+28
		+2

Standard otonal notation below ♮A. The notation shows a series of notes on a staff, with microtonal accidentals (numbers) below them. The notes are labeled with intervals and ratios.

Interval	Ratio	Microtonal Accidental
m7	(256:261)	-27
P8	(32:31)	+30
M2	(36:37)	-12
M3	(81:82)	+5
P4	(128:129)	+4
aug4	(729:752)	+51
		+29
		+12
		+51
		+28
		E-34
		+2
		E+38

Standard utonal notation below ♮E

Standard utonal notation below ♮E. The notation shows a series of notes on a staff, with microtonal accidentals (numbers) below them. The notes are labeled with intervals and ratios.

Interval	Ratio	Microtonal Accidental
M3	(80:81)	+2
m7	(63:64)	+16
P4	(33:32)	+33
M6	(26:27)	+2
aug8	(2176:2187)	+16
m3	(513:512)	+31
aug4	(736:729)	+4
		+16
		+31

Standard utonal notation below ♮E. The notation shows a series of notes on a staff, with microtonal accidentals (numbers) below them. The notes are labeled with intervals and ratios.

Interval	Ratio	Microtonal Accidental
m7	(261:256)	+29
P8	(31:32)	G#-39
M2	(37:36)	+33
M3	(82:81)	+14
P4	(129:128)	F-43
aug4	(752:729)	+2
		+47
		+4
		C#-41
		+16
		+31
		+12



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Standard otonal notation above ♭A

partial interval alteration (81:80) 5° M3 (81:80) 7° m7 (64:63) 11° P4 (32:33) 13° M6 (27:26) 17° aug8 (2187:2176) 19° m3 (512:513) 23° aug4 (729:736)

Standard utonal notation below ♭E

u5 M3 (80:81) u7 m7 (63:64) u11 P4 (33:32) u13 M6 (26:27) u17 aug8 (2176:2187) u19 m3 (513:512) u23 aug4 (736:729) u29 m7 (261:256) u31 P8 (31:32) u37 M2 (37:36) u41 M3 (82:81) u43 P4 (129:128) u47 aug4 (752:729)



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Standard otonal notation above ♮A

Standard otonal notation above ♮A. The notation shows a series of notes on a staff, with various microtonal accidentals and cents indications. The notes are labeled with their corresponding intervals and ratios.

Interval	Ratio	Cents
partial interval alteration		
5°	(81:80)	-14
7°	(64:63)	-31
11°	(32:33)	-14
13°	(27:26)	-12
17°	(2187:2176)	-2
19°	(512:513)	-14
23°	(729:736)	-29
29°	(256:261)	-12
31°	(32:31)	-45
37°	(36:37)	-2
41°	(81:82)	-29
43°	(128:129)	-10
47°	(729:752)	-10

Standard utonal notation below ♮E

Standard utonal notation below ♮E. The notation shows a series of notes on a staff, with various microtonal accidentals and cents indications. The notes are labeled with their corresponding intervals and ratios.

Interval	Ratio	Cents
u5	(80:81)	+2
u7	(63:64)	+16
u11	(33:32)	-49
u13	(26:27)	-2
u17	(2176:2187)	-3
u19	(513:512)	-2
u23	(736:729)	-26
u29	(261:256)	-28
u31	(31:32)	-51
u37	(37:36)	-2
u41	(82:81)	-27
u43	(129:128)	-10
u47	(752:729)	-26



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Standard orton notation above ♮A

Standard orton notation above ♮A. The notation shows a series of notes on a staff, with various microtonal accidentals and ratios indicated below the notes. The notes are labeled with their corresponding ratios and intervals.

Note	Ratio	Interval
8 ₁		
partial interval alteration		
5°	(81:80)	M3
7°	(64:63)	m7
11°	(32:33)	P4
13°	(27:26)	M6
17°	(2187:2176)	aug8
19°	(512:513)	m3
23°	(729:736)	aug4

Standard orton notation below ♮E. The notation shows a series of notes on a staff, with various microtonal accidentals and ratios indicated below the notes. The notes are labeled with their corresponding ratios and intervals.

Note	Ratio	Interval
u5	(80:81)	M3
u7	(63:64)	m7
u11	(33:32)	P4
u13	(26:27)	M6
u17	(2176:2187)	aug8
u19	(513:512)	m3
u23	(736:729)	aug4



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Standard otonal notation above ♮A

Standard otonal notation above ♮A. The notation shows a series of notes on a staff, with microtonal accidentals (numbers) above them. The notes are labeled with intervals and ratios. The notes are: 8[♭], partial interval alteration, 5[°] (81:80), 7[°] (64:63), 11[°] (32:33), 13[°] (27:26), 17[°] (2187:2176), 19[°] (512:513), 23[°] (729:736), 29[°] (256:261), 31[°] (32:31), 37[°] (36:37), 41[°] (81:82), 43[°] (128:129), 47[°] (729:752), and E+38. The notes are grouped into pairs, with the first note of each pair being a Pythagorean note and the second note being a modified Pythagorean note. The modified notes are labeled with microtonal accidentals (numbers) above them.

Standard utonal notation below ♮E

Standard utonal notation below ♮E. The notation shows a series of notes on a staff, with microtonal accidentals (numbers) below them. The notes are labeled with intervals and ratios. The notes are: 8[♭], 5[♭] (80:81), 7[♭] (63:64), 11[♭] (33:32), 13[♭] (26:27), 17[♭] (2176:2187), 19[♭] (513:512), 23[♭] (736:729), 29[♭] (261:256), 31[♭] (31:32), 37[♭] (37:36), 41[♭] (82:81), 43[♭] (129:128), 47[♭] (752:729), and A+36. The notes are grouped into pairs, with the first note of each pair being a Pythagorean note and the second note being a modified Pythagorean note. The modified notes are labeled with microtonal accidentals (numbers) below them.



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Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

Notes with “normal” accidentals are tuned in pure fifths

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
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building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz*

Standard otonal notation above ♭A

Standard otonal notation above ♭A. The notation shows a series of notes on a staff, with microtonal accidentals and cents indications. The notes are labeled with their corresponding ratios and names.

Note	Ratio	Cents
♭A	8 ¹	-27
F ⁺ 41		F ⁺ 41
♭A		-31
29° m7	(256:261)	-12
A [♭] +45		A [♭] +45
31° P8	(32:31)	-45
A [♯] -47		-45
37° M2	(36:37)	-2
C+42		-14
41° M3	(81:82)	-29
43° P4	(128:129)	-10
47° aug4	(729:752)	

Standard utonal notation below ♭E

Standard utonal notation below ♭E. The notation shows a series of notes on a staff, with microtonal accidentals and cents indications. The notes are labeled with their corresponding ratios and names.

Note	Ratio	Cents
♭E	8 ¹	+2
G [♯] -39		G [♯] -39
♭E		+2
u5 M3	(80:81)	+16
u7 m7	(63:64)	+33
u11 P4	(33:32)	+2
u13 M6	(26:27)	+16
u17 aug8	(2176:2187)	-3
u19 m3	(513:512)	-2
u23 aug4	(736:729)	-49
u29 m7	(261:256)	-28
u31 P8	(31:32)	-51
u37 M2	(37:36)	-2
u41 M3	(82:81)	-27
u43 P4	(129:128)	-10
u47 aug4	(752:729)	-26



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Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

partial 5° and its multiples are indicated by an arrow

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in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above ♮A

8₁ partial interval alteration 5° M3 (81:80) 7° m7 (64:63) 11° P4 (32:33) 13° M6 (27:26) 17° aug8 (2187:2176) 19° m3 (512:513) 23° aug4 (729:736)

29° m7 (256:261) 31° P8 (32:31) 37° M2 (36:37) 41° M3 (81:82) 43° P4 (128:129) 47° aug4 (729:752)

Standard utonal notation below ♮E

in the subharmonic row, the same intervals appear, but are *inverted*

u5 M3 (80:81) u7 m7 (63:64) u11 P4 (33:32) u13 M6 (26:27) u17 aug8 (2176:2187) u19 m3 (513:512) u23 aug4 (736:729)

u29 m7 (261:256) u31 P8 (31:32) u37 M2 (37:36) u41 M3 (82:81) u43 P4 (129:128) u47 aug4 (752:729)



The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

partial 7° and its multiples (septimal notes) are indicated by Tartini's hook revised by Marc Sabat & Thomas Nicholson

*in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz*

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above ♭A

Musical notation for Standard otonal notation above ♭A. The notation is presented on two staves. The first staff (bass clef) shows notes with microtonal accidentals and ratios. The second staff (treble clef) shows notes with microtonal accidentals and ratios. The notes are labeled with their prime numbers and ratios. The notes are: 8^u, partial interval alteration, 5° (81:80), 7° (64:63), 11° (32:33), 13° (27:26), 17° (2187:2176), 19° (512:513), 23° (729:736), 29° (256:261), 31° (32:31), 37° (36:37), 41° (81:82), 43° (128:129), 47° (729:752), and E+38. The notes 7° and E+38 are highlighted with a yellow background.

Standard utonal notation below ♭E

Musical notation for Standard utonal notation below ♭E. The notation is presented on two staves. The first staff (treble clef) shows notes with microtonal accidentals and ratios. The second staff (bass clef) shows notes with microtonal accidentals and ratios. The notes are labeled with their prime numbers and ratios. The notes are: 5^u (80:81), 7^u (63:64), 11^u (33:32), 13^u (26:27), 17^u (2176:2187), 19^u (513:512), 23^u (736:729), 29^u (261:256), 31^u (31:32), 37^u (37:36), 41^u (82:81), 43^u (129:128), 47^u (752:729), and A-36. The notes 7^u and A-36 are highlighted with a yellow background.



The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

partial 9° is *odd* (new pitch-class) but it isn't *prime*...

revised by Marc Sabat & Thomas Nicholson

*in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz*

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above ♭A

Musical notation for Standard otonal notation above ♭A. The notation is written on a bass clef staff. The notes are: 8[♭], partial interval alteration, 5° (81:80), 7° (64:63), 11° (32:33), 13° (27:26), 17° (2187:2176), 19° (512:513), 23° (729:736). The notes are marked with various microtonal accidentals and cents values: +2, -14, +2, -31, +4, -14, +51, +2, +5, +4, +51, +28, +2. A yellow highlight is placed over the 11° note.

Standard utonal notation below ♭E

Musical notation for Standard utonal notation below ♭E. The notation is written on a treble clef staff. The notes are: u5 (80:81), u7 (63:64), u11 (33:32), u13 (26:27), u17 (2176:2187), u19 (513:512), u23 (736:729). The notes are marked with various microtonal accidentals and cents values: +2, +2, +16, +33, +2, +16, -2, -49, +16, G#-39, +33, +14, +2, +47, +4, C#-41, +16, +31, +12, -26, -49, -26, A+36, A-36. A yellow highlight is placed over the u11 note.



The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

undecimal notes:

partial 11° and its multiples are indicated by a 2-stroke cross

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above ♮A

Standard otonal notation above ♮A. The notation shows a series of notes on a staff, with microtonal accidentals (2-stroke crosses) indicating deviations from the Pythagorean scale. The notes are labeled with ratios and cents deviations.

Note	Ratio	Cents Deviation
8 ₁		
partial interval alteration		
5°	M3 (81:80)	-14
7°	m7 (64:63)	-31
11°	P4 (32:33)	+51
13°	M6 (27:26)	-12
17°	aug8 (2187:2176)	+5
19°	m3 (512:513)	-2
23°	aug4 (729:736)	+28
29°	m7 (256:261)	-12
31°	P8 (32:31)	+5
37°	M2 (36:37)	+4
41°	M3 (81:82)	+29
43°	P4 (128:129)	+12
47°	aug4 (729:752)	+2

Standard utonal notation below ♮E

Standard utonal notation below ♮E. The notation shows a series of notes on a staff, with microtonal accidentals (2-stroke crosses) indicating deviations from the Pythagorean scale. The notes are labeled with ratios and cents deviations.

Note	Ratio	Cents Deviation
u5	M3 (80:81)	+16
u7	m7 (63:64)	+33
u11	P4 (33:32)	-49
u13	M6 (26:27)	+33
u17	aug8 (2176:2187)	-3
u19	m3 (513:512)	-2
u23	aug4 (736:729)	+31
u29	m7 (261:256)	-28
u31	P8 (31:32)	+14
u37	M2 (37:36)	+4
u41	M3 (82:81)	+31
u43	P4 (129:128)	+12
u47	aug4 (752:729)	+2

... or a reversed flat



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Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

tridecimal notes:

partial 13° and its multiples are indicated by a 3-stroke reverse flat

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Standard otonal notation above ♮A

Musical notation for standard otonal series above A. The notation is written on a grand staff (bass and treble clefs). The notes are modified with various symbols (flats, sharps, and a 3-stroke reverse flat) to represent different intervals. The intervals are labeled with their ratios and degrees. The 13th partial is highlighted in yellow.

Interval	Ratio	Cent Deviation
partial interval alteration		-27
8 ^u		+2
5°	(81:80)	-14
7°	(64:63)	-31
11°	(32:33)	+51
13°	(27:26)	-31
17°	(2187:2176)	+5
19°	(512:513)	+4
23°	(729:736)	+51
29°	(256:261)	+6
31°	(32:31)	+30
37°	(36:37)	+5
41°	(81:82)	+29
43°	(128:129)	+12
47°	(729:752)	+28

Standard utonal notation below ♮E

Musical notation for standard utonal series below E. The notation is written on a grand staff (treble and bass clefs). The notes are modified with various symbols (flats, sharps, and a 3-stroke reverse flat) to represent different intervals. The intervals are labeled with their ratios and degrees. The 13th partial is highlighted in yellow.

Interval	Ratio	Cent Deviation
u5	(80:81)	+2
u7	(63:64)	+16
u11	(33:32)	+16
u13	(26:27)	+33
u17	(2176:2187)	+2
u19	(513:512)	+4
u23	(736:729)	+31
u29	(261:256)	+29
u31	(31:32)	+33
u37	(37:36)	+47
u41	(82:81)	+33
u43	(129:128)	+16
u47	(752:729)	+12



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Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

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Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
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Standard otonal notation above ♭A

as the row approaches 16°, the steps become semitones

8¹ partial interval alteration +2 -14 5° M3 (81:80) +2 -31 7° m7 (64:63) +4 -14 11° P4 (32:33) +51 13° M6 (27:26) +2 F+41 -31 -12 17° aug8 (2187:2176) +5 +4 19° m3 (512:513) +51 +28 23° aug4 (729:736) +2

-27 F+41 -31 29° m7 (256:261) +6 +30 31° P8 (32:31) -12 A♭+45 A♯-47 -45 37° M2 (36:37) +5 +4 +51 -2 C+42 -14 41° M3 (81:82) +29 -29 43° P4 (128:129) +12 +51 -10 47° aug4 (729:752) +28 E-34 +2 E+38

Standard utonal notation below ♭E

+2 8¹ +2 +16 +33 +2 -2 11° P4 (33:32) +16 G♯-39 +33 +14 13° M6 (26:27) +2 F-43 +47 17° aug8 (2176:2187) +31 +4 +16 +31 19° m3 (513:512) -49 -26 23° aug4 (736:729) +29 G♯-39 +33 -4 -28 -51 -3 -2 -49 -27 41° M3 (82:81) -10 -49 43° P4 (129:128) -26 A+36 47° aug4 (752:729) A-36



with dedicated microtonal accidental symbols for primes 5 through 47

The harmonic partial row is a source for intervals used in music:

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Standard otonal notation above $\sharp A$

Standard utonal notation below ♯E

8₇ partial interval alteration

5° M3 (81:80)

7° m7 (64:63)

11° P4 (32:33)

13° M6 (27:26)

17° aug8 (2187:2176)

19° m3 (512:513)

23° aug4 (729:736)

29° m7 (256:261)

31° P8 (32:31)

37° M2 (36:37)

41° M3 (81:82)

43° P4 (128:129)

47° aug4 (729:752)

u5 M3 (80:81)

u7 m7 (63:64)

u11 P4 (33:32)

u13 M6 (26:27)

u17 aug8 (2176:2187)

u19 m3 (513:512)

u23 aug4 (736:729)

u29 m7 (261:256)

u31 P8 (31:32)

u37 M2 (37:36)

u41 M3 (82:81)

u43 P4 (129:128)

u47 aug4 (752:729)

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Standard otonal notation above ♭A

an Octave with harmonic ratio 1:2

Partial	Interval	Ratio	Cents
8 ^u	partial interval alteration		
5 ^o	M3	(81:80)	-14
7 ^o	m7	(64:63)	-31
11 ^o	P4	(32:33)	-14
13 ^o	M6	(27:26)	-31
17 ^o	aug8	(2187:2176)	-12
19 ^o	m3	(512:513)	-2
23 ^o	aug4	(729:736)	-14
29 ^o	m7	(256:261)	-29
31 ^o	P8	(32:31)	-12
37 ^o	M2	(36:37)	-45
41 ^o	M3	(81:82)	-2
43 ^o	P4	(128:129)	-14
47 ^o	aug4	(729:752)	-10

Standard utonal notation below ♭E

Partial	Interval	Ratio	Cents
u5	M3	(80:81)	+2
u7	m7	(63:64)	+16
u11	P4	(33:32)	+33
u13	M6	(26:27)	+2
u17	aug8	(2176:2187)	-49
u19	m3	(513:512)	-3
u23	aug4	(736:729)	-2
u29	m7	(261:256)	+31
u31	P8	(31:32)	+4
u37	M2	(37:36)	+16
u41	M3	(82:81)	+3
u43	P4	(129:128)	-27
u47	aug4	(752:729)	-10

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Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

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Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above ♮A

8¹ partial interval alteration

+2 -14 -31 -14 +4 +51 +2 -31 -12 +5 +4 -2 -14 -29 +51 +28 +2

5° M3 (81:80) 7° m7 (64:63) 11° P4 (32:33) 13° M6 (27:26) 17° aug8 (2187:2176) 19° m3 (512:513) 23° aug4 (729:736)

29° m7 (256:261) 31° P8 (32:31) 37° M2 (36:37) 41° M3 (81:82) 43° P4 (128:129) 47° aug4 (729:752)

-27 F+41 -31 -12 A♭+45 A♯-47 -45 -2 C+42 -14 -29 -10

Standard utonal notation below ♮E

+2 8¹ +2 +16 +33 +2 +16 G♯-39 +33 +14 +2 +4 +16 +31

u5 M3 (80:81) u7 m7 (63:64) u11 P4 (33:32) u13 M6 (26:27) u17 aug8 (2176:2187) u19 m3 (513:512) u23 aug4 (736:729)

+29 G♯-39 +33 +14 F-43 +2 +47 +4 C♯-41 +16 +31 +12

u29 m7 (261:256) u31 P8 (31:32) u37 M2 (37:36) u41 M3 (82:81) u43 P4 (129:128) u47 aug4 (752:729)

-4 -28 -51 -3 -2 -49 -27 -10 -49 -26 A+36 A-36

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Standard otonal notation above ♮A

Fourth 3:4

Interval	Ratio	Cents Deviation
partial interval alteration		-27
8 ^u		+2
5 ^o	(81:80)	-14
M3		+2
m7	(64:63)	-31
F+41		+41
11 ^o	(32:33)	-14
P4		+51
M6	(27:26)	-12
aug8	(2187:2176)	+5
m3	(512:513)	+4
aug4	(729:736)	+51
29 ^o	(256:261)	+6
m7		+30
31 ^o	(32:31)	+5
P8		+4
M2	(36:37)	+51
41 ^o	(81:82)	+29
M3		+12
43 ^o	(128:129)	+51
P4		+28
47 ^o	(729:752)	E-34
aug4		+2
E+38		+38

Standard utonal notation below ♮E

Interval	Ratio	Cents Deviation
u5	(80:81)	+2
M3		+2
u7	(63:64)	+16
m7		+33
P4	(33:32)	-2
M6	(26:27)	+16
aug8	(2176:2187)	G#-39
m3	(513:512)	+33
aug4	(736:729)	+14
u29	(261:256)	+29
m7		G#-39
u31	(31:32)	+33
P8		+14
u37	(37:36)	F-43
M2		+2
u41	(82:81)	+47
M3		+4
u43	(129:128)	C#-41
P4		+16
u47	(752:729)	+31
aug4		+12
A+36		-4
A-36		-28
u29		-51
u31		-3
u37		-2
u41		-49
u43		-27
u47		-10
aug4		-49
aug4		-26
aug4		-26

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Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

and the row also contains chords, i.e. the Major Triad 4:5:6

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above ♮A

Musical notation for Standard otonal notation above ♮A. The notation is presented on two staves. The first staff (bass clef) shows notes with microtonal accidentals and cents deviations. The second staff (treble clef) shows notes with microtonal accidentals and cents deviations. A speaker icon is located between the two staves.

Interval	Ratio	Cents Deviation
partial interval alteration		-27
8 ^u		+2
5 ^o	(81:80)	-14
7 ^o	(64:63)	-31
11 ^o	(32:33)	-14
P4		+51
M6	(27:26)	+2
aug8	(2187:2176)	+5
m3	(512:513)	+4
aug4	(729:736)	+51
29 ^o	(256:261)	+6
m7		+30
P8	(32:31)	+5
M2	(36:37)	+4
M3	(81:82)	+29
P4	(128:129)	+12
aug4	(729:752)	+51
47 ^o		+28
		E-34
		+2
		E+38

Standard utonal notation below ♮E

Musical notation for Standard utonal notation below ♮E. The notation is presented on two staves. The first staff (treble clef) shows notes with microtonal accidentals and cents deviations. The second staff (bass clef) shows notes with microtonal accidentals and cents deviations.

Interval	Ratio	Cents Deviation
u5	(80:81)	+2
M3		+2
u7	(63:64)	+16
m7		+33
P4	(33:32)	+2
M6	(26:27)	+16
aug8	(2176:2187)	-2
m3	(513:512)	-3
aug4	(736:729)	-2
u23		+4
u19		+16
u17		+31
u13		-49
u11		-26
P4		-49
M2	(37:36)	-2
M3	(82:81)	-49
P4	(129:128)	-27
aug4	(752:729)	-10
u29	(261:256)	-4
m7		-28
P8	(31:32)	-51
u31		-3
u37		-2
u41		-49
u43		-26
u47		-49
u29		+29
u31		G#-39
u37		+33
u41		+14
u43		F-43
u47		+2
u29		+47
u31		+4
u37		C#-41
u41		+16
u43		+31
u47		+12

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

revised by Marc Sabat & Thomas Nicholson

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Standard otonal notation above ♭A

partial interval alteration +2

81:80

M3

81:80

The image shows a musical score for the song "The Sound of Silence" by Simon & Garfunkel. The score is written for guitar and piano. The guitar part is on the top staff, and the piano part is on the bottom staff. The score includes various chords and intervals, such as M3, m7, P4, M6, aug8, m3, and aug4. The score is labeled with "Guitar" and "Piano" and includes a key signature of one sharp (F#).

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Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

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Minor Triad 10:12:15

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Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above ♭A

8_♭ partial interval alteration

+2 -14 -31 -14 +4 +51 +2 -31 -12 +5 +4 -2 -14 -29 +51 +28 +2

5° M3 (81:80) 7° m7 (64:63) 11° P4 (32:33) 13° M6 (27:26) 17° aug8 (2187:2176) 19° m3 (512:513) 23° aug4 (729:736)

-27 F+41 -31 -12 A♭+45 A♯-47 -45 37° M2 (36:37) -2 C+42 -14 41° M3 (81:82) 43° P4 (128:129) -10 47° aug4 (729:752)

+6 +30 +5 +4 +51 +29 +12 +51 +28 E-34 +2 E+38

Standard utonal notation below ♭E

+2 8_♭ +2 +16 +33 +2 +16 G♯-39 +33 +14 +2 +4 +16 +31

u5 M3 (80:81) u7 m7 (63:64) u11 P4 (33:32) u13 M6 (26:27) u17 aug8 (2176:2187) u19 m3 (513:512) u23 aug4 (736:729)

+29 G♯-39 +33 +14 F-43 +2 +47 +4 C♯-41 +16 +31 +12

-4 -28 -51 -3 -2 -49 -27 -10 -49 -26 A+36 A-36

u29 m7 (261:256) u31 P8 (31:32) u37 M2 (37:36) u41 M3 (82:81) u43 P4 (129:128) u47 aug4 (752:729)

The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

Diminished Triad 5:6:7

revised by Marc Sabat & Thomas Nicholson

*in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz*

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above ♮A

8[♭] partial interval alteration +2 -14 5° M3 (81:80) +2 -31 7° m7 (64:63) +4 -14 11° P4 (32:33) +51 13° M6 (27:26) +5 +4 17° aug8 (2187:2176) -2 -14 19° m3 (512:513) -29 +51 +28 23° aug4 (729:736) +2

-27 F+41 -31 29° m7 (256:261) +30 -12 A♭+45 31° P8 (32:31) A♯-47 -45 37° M2 (36:37) +51 -2 C+42 -14 41° M3 (81:82) +29 -29 43° P4 (128:129) +12 -10 47° aug4 (729:752) +51 +28 E-34 +2 E+38

Standard utonal notation below ♮E

+2 8[♭] +2 +16 +33 +2 +16 u5 M3 (80:81) +33 +2 u7 m7 (63:64) -2 -49 u11 P4 (33:32) G♯-39 +33 +14 u13 M6 (26:27) +2 -3 u17 aug8 (2176:2187) -2 u19 m3 (513:512) +16 +31 u23 aug4 (736:729) -49 -26

+29 G♯-39 +33 +14 F-43 +2 +47 +4 C♯-41 +16 +31 +12 u29 m7 (261:256) u31 P8 (31:32) u37 M2 (37:36) -27 u41 M3 (82:81) -10 u43 P4 (129:128) -49 -26 A+36 u47 aug4 (752:729) A-36

The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

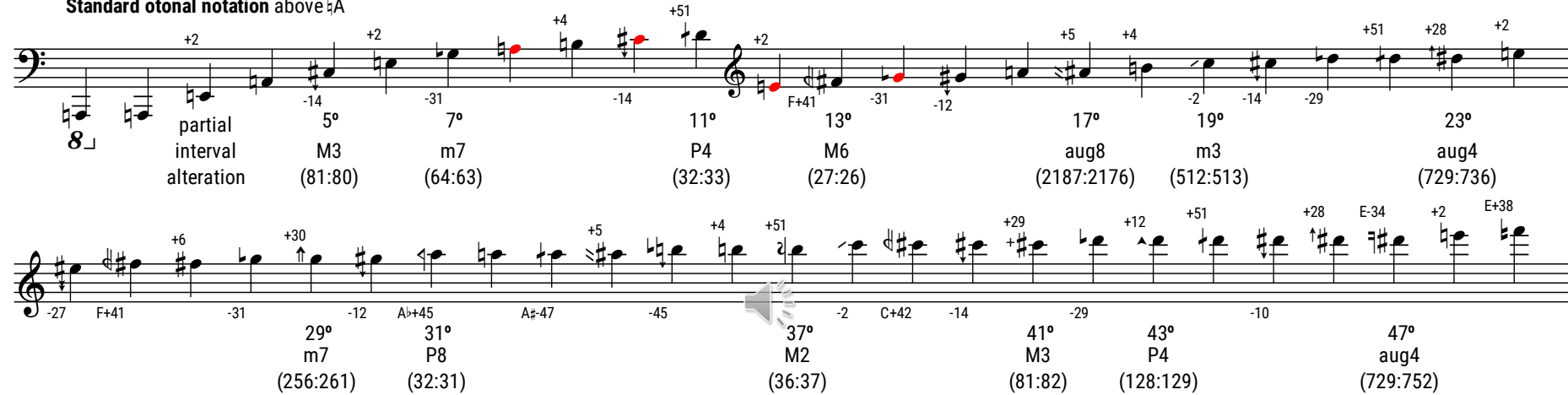
Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

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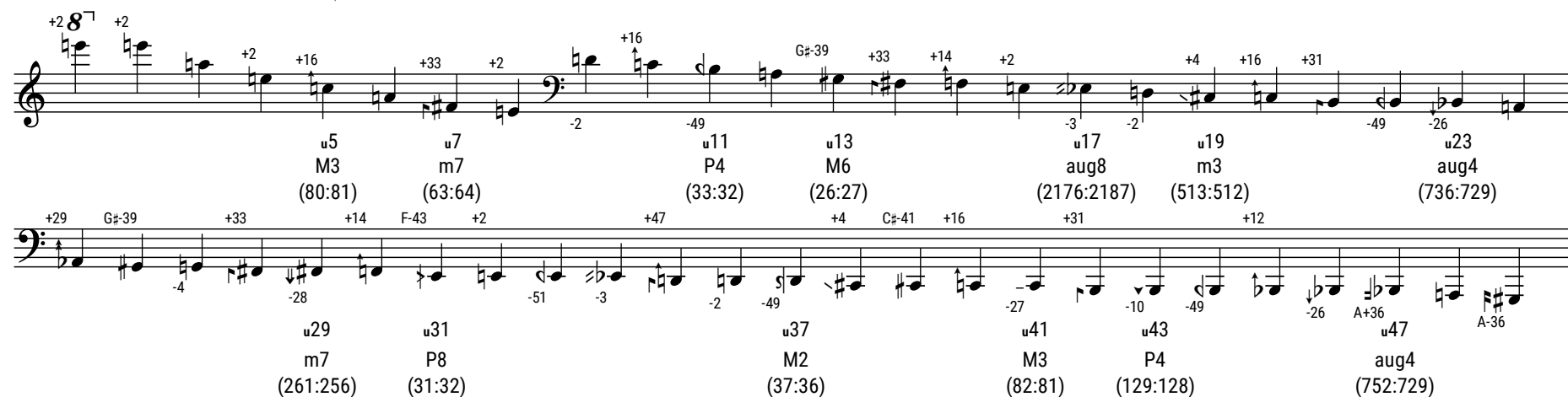
and aggregates of four or more notes, i.e. the Dominant 7th

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above ♮A



Standard utonal notation below ♮E



Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

revised by Marc Sabat & Thomas Nicholson

in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJII devised by Marc Sabat and Wolfgang von Schweinitz

Standard otonal notation above $\sharp A$

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Diminished Chord

revised by Marc Sabat & Thomas Nicholson

*in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz*

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above ♮A

partial interval alteration

8

5° (81:80)

7° (64:63)

11° (32:33)

13° (27:26)

17° (2187:2176)

19° (512:513)

23° (729:736)

29° (256:261)

31° (32:31)

37° (36:37)

41° (81:82)

43° (128:129)

47° (729:752)

Standard utonal notation below ♮E

u5 (80:81)

u7 (63:64)

u11 (33:32)

u13 (26:27)

u17 (2176:2187)

u19 (513:512)

u23 (736:729)

u29 (261:256)

u31 (31:32)

u37 (37:36)

u41 (82:81)

u43 (129:128)

u47 (752:729)

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Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

and new microtonal chords, i.e. Septimal Chord 6:7:8:9

revised by Marc Sabat & Thomas Nicholson

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building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz*

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above ♮A

musical notation for standard otonal series above A, showing intervals and ratios:

Interval	Ratio	Cents
partial interval		
8 ^u		+2
M3	(81:80)	-14
m7	(64:63)	-31
P4	(32:33)	+51
M6	(27:26)	+2
aug8	(2187:2176)	+5
m3	(512:513)	+4
aug4	(729:736)	+51
		+28
		+2

musical notation for standard otonal series below A, showing intervals and ratios:

Interval	Ratio	Cents
29 ^o	(256:261)	-27
m7		F+41
		-31
		-12
31 ^o	(32:31)	A♭+45
P8		A♯-47
		-45
37 ^o	(36:37)	+51
M2		-2
		C+42
		-14
41 ^o	(81:82)	-29
M3		+29
		+12
43 ^o	(128:129)	+51
P4		+28
		-10
47 ^o	(729:752)	E-34
aug4		+2
		E+38

Standard utonal notation below ♮E

musical notation for standard utonal series below E, showing intervals and ratios:

Interval	Ratio	Cents
u5	(80:81)	+2
M3		+2
u7	(63:64)	+16
m7		+33
		+2
u11	(33:32)	-2
P4		-49
u13	(26:27)	G♯-39
M6		+33
		+14
u17	(2176:2187)	+2
aug8		-3
		-2
u19	(513:512)	+4
m3		+16
		+31
u23	(736:729)	-49
aug4		-26

musical notation for standard utonal series below E, showing intervals and ratios:

Interval	Ratio	Cents
u29	(261:256)	+29
m7		G♯-39
		+33
u31	(31:32)	+14
P8		F-43
		+2
u37	(37:36)	+47
M2		+4
		C♯-41
		+16
u41	(82:81)	+31
M3		+12
		-27
u43	(129:128)	-10
P4		-49
		-26
u47	(752:729)	A+36
aug4		A-36

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compare the Septimal Chord 6:7:8:9 an octave lower

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Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above ♭A

8[♭] partial interval alteration

+2 -14 -31 -14 +4 +51 +2 -31 -12 +5 +4 -2 -14 -29 +51 +28 +2

5° M3 (81:80) 7° m7 (64:63) 11° P4 (32:33) 13° M6 (27:26) 17° aug8 (2187:2176) 19° m3 (512:513) 23° aug4 (729:736)

-27 F+41 -31 -12 A♭+45 A♯-47 -45 37° M2 (36:37) -2 C+42 -14 -29 41° M3 (81:82) 43° P4 (128:129) -10 47° aug4 (729:752)

+6 +30 +5 +4 +51 +29 +12 +51 +28 E-34 +2 E+38

Standard utonal notation below ♭E

+2 8[♭] +2 +16 +33 +2 +16 G♯-39 +33 +14 +2 +47 +4 C♯-41 +16 +31 +12

u5 M3 (80:81) u7 m7 (63:64) u11 P4 (33:32) u13 M6 (26:27) u17 aug8 (2176:2187) u19 m3 (513:512) u23 aug4 (736:729)

+29 G♯-39 +33 +14 F-43 +2 -49 -2 -49 -26 A+36 A-36

u29 m7 (261:256) u31 P8 (31:32) u37 M2 (37:36) u41 M3 (82:81) u43 P4 (129:128) u47 aug4 (752:729)

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

revised by Marc Sabat & Thomas Nicholson

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building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz*

Standard notation above ♯A

8

partial interval alteration

5° (81:80)

7° (64:63)

11° P4 (32:33)

13° M6 (27:26)

17° aug8 (2187:2176)

19° m3 (512:513)

23° aug4 (729:736)

29° m7 (256:261)

31° P8 (32:31)

37° M2 (36:37)

41° M3 (81:82)

43° P4 (128:129)

47° aug4 (729:752)

[illegible]



The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

Tridecimal Chord 10:12:13:15

revised by Marc Sabat & Thomas Nicholson

in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above ♮A

8, partial interval alteration, 5° (81:80), 7° (64:63), 11° (32:33), 13° (27:26), 17° (2187:2176), 19° (512:513), 23° (729:736), 29° (256:261), 31° (32:31), 37° (36:37), 41° (81:82), 43° (128:129), 47° (729:752)

Standard utonal notation below ♮E

u5 (80:81), u7 (63:64), u11 (33:32), u13 (26:27), u17 (2176:2187), u19 (513:512), u23 (736:729), u29 (261:256), u31 (31:32), u37 (37:36), u41 (82:81), u43 (129:128), u47 (752:729)



Harmony by numbers: tuning in practice

- How can a performer learn to hear and play such tunings accurately?



Harmony by numbers: tuning in practice

- How can a performer learn to hear and play such tunings accurately?
by perceiving acoustic and psychoacoustic phenomena



Harmony by numbers: difference (Tartini) tones

When two frequencies interact in the same medium (air, eardrum, brain, etc.) they produce *combination tones*, i.e., distortion at various linear combinations of their frequencies:

$$m \cdot F_1 + n \cdot F_2$$

where m and n are integers. The combination tone most easily perceived is the *difference tone*.



Harmony by numbers: difference (Tartini) tones

When two frequencies interact in the same medium (air, eardrum, brain, etc.) they produce *combination tones*, i.e., distortion at various linear combinations of their frequencies:

$$m \cdot F_1 + n \cdot F_2$$

where m and n are integers. The combination tone most easily perceived is the *difference tone*.

In many simple ratios between partials, like 1:2 (octave), 2:3 (fifth), 3:4 (fourth), 4:5 (major third), and 3:5 (major sixth), the difference tone reinforces their common fundamental, making a clear, consonant harmony.

Harmony by numbers: difference (Tartini) tones

Violin

8/5 13/8 5/3

3° 5° 2°

Difference Tones
&
Fundamentals

The image displays a musical score for a violin and a section for difference tones and fundamentals. The violin part consists of three measures, each containing a dyad of notes. Above each measure is a box with a ratio: 8/5, 13/8, and 5/3. The first measure has notes on the second and fourth lines of the staff. The second measure has notes on the first and third lines. The third measure has notes on the second and fourth lines. The difference tones and fundamentals section consists of three measures. Above each measure is a box with a degree: 3°, 5°, and 2°. The first measure has notes on the second and fourth lines. The second measure has notes on the first and third lines. The third measure has notes on the second and fourth lines. The fundamentals are shown in the bass clef, with notes on the second and fourth lines. The difference tones are shown in the treble clef, with notes on the second and fourth lines.



Harmony by numbers: common partials

When two frequencies are tuned rationally as $a : b$, expressed in lowest terms, they will share some common partials. Namely, the b th partial of a is equal to the a th partial of b . The lowest common partial of the two pitches may be calculated as the *product* of a and b .

$$\text{Common partial} = a \cdot b$$



Harmony by numbers: tuning by listening to beating

When two frequencies are tuned rationally as $a : b$, expressed in lowest terms, they will share some common partials. Namely, the b th partial of a is equal to the a th partial of b . The lowest common partial of the two pitches may be calculated as the *product* of a and b .

$$\text{Common partial} = a \cdot b$$

Beating at the common partial is one of the most commonly perceived phenomena when musicians learn to tune intervals. Eliminating beating to produce so-called “pure” intervals is one way to make JI sounds.

Harmony by numbers: tuning by listening to beating

Common Partial

Violin

$5/3$

5°

3°

The diagram illustrates a musical tuning exercise. It consists of two staves: 'Common Partial' (top) and 'Violin' (bottom). The 'Common Partial' staff shows three notes: a half note on the first line (C4), a half note on the second line (E4), and a half note on the second space (G4). A bracket above the first two notes is labeled 5° , and a bracket above the last two notes is labeled 3° . The 'Violin' staff shows a sequence of notes: a half note on the first line (C4), a half note on the first space (D4), a half note on the second line (E4), a half note on the second space (G4), a half note on the third line (B4), and a half note on the third space (D5). A bracket above the first two notes is labeled $5/3$. The notes are written in treble clef. The diagram is used to demonstrate how to tune the violin by listening to the beating of the intervals.



Harmony by numbers: degrees of tuneability

Small number ratios produce intervals that may be easily identified and tuned. As numbers comprising a ratio become larger, pitch distances to the interval's common fundamental and common partial also increase. Eventually, such intervals are no longer precisely recognisable, and therefore, no longer tuneable by ear.



Harmony by numbers: chords, melodies

Small number ratios produce intervals that may be easily identified and tuned. As numbers comprising a ratio become larger, pitch distances to the interval's common fundamental and common partial also increase. Eventually, such intervals are no longer precisely recognisable, and therefore, no longer tuneable by ear.

In my music for live musicians, I find it useful to work with tuneability, to focus on the experience of hearing and playing the unique sonorities and resonances of rational intonation. Concentrating on intervals and chords making melodies by means of counterpoint, rather than using fixed scales or a single row, allows a free flow of sounds with shifting fundamentals and changing points of reference.



Harmony by numbers: applying JI to music repertoire

- tuning of the open strings



Harmony by numbers: applying JI to music repertoire

- tuning of the open strings *in perfect fifths*



Harmony by numbers: applying JI to music repertoire

- tuning of the open strings *in perfect fifths*
- tuning of major, minor, and diminished chords



Harmony by numbers: applying JI to music repertoire

- tuning of the open strings *in perfect fifths*
- tuning of major, minor, and diminished chords
1:3:5 (major)



Harmony by numbers: applying JI to music repertoire

- tuning of the open strings *in perfect fifths*
- tuning of major, minor, and diminished chords
1:3:5 (major); 3:5:15 (minor)



Harmony by numbers: applying JI to music repertoire

- tuning of the open strings *in perfect fifths*
- tuning of major, minor, and diminished chords
1:3:5 (major); 3:5:15 (minor); 3:5:7 (diminished)



Harmony by numbers: applying JI to music repertoire

- tuning of the open strings *in perfect fifths*
- tuning of major, minor, and diminished chords
1:3:5 (major); 3:5:15 (minor); 3:5:7 (diminished)
- tuning of scales and modes



Harmony by numbers: applying JI to music repertoire

- tuning of the open strings *in perfect fifths*
- tuning of major, minor, and diminished chords
1:3:5 (major); 3:5:15 (minor); 3:5:7 (diminished)
- tuning of scales and modes, *major*



Harmony by numbers: applying JI to music repertoire

- tuning of the open strings *in perfect fifths*
- tuning of major, minor, and diminished chords
1:3:5 (major); 3:5:15 (minor); 3:5:7 (diminished)
- tuning of scales and modes, *major and minor*



Harmony by numbers: applying JI to music repertoire

- tuning of the open strings *in perfect fifths*
- tuning of major, minor, and diminished chords
1:3:5 (major); 3:5:15 (minor); 3:5:7 (diminished)
- tuning of scales and modes, *major and minor*
- modulation?



Harmony by numbers: applying JI to music repertoire

- tuning of the open strings *in perfect fifths*
- tuning of major, minor, and diminished chords
1:3:5 (major); 3:5:15 (minor); 3:5:7 (diminished)
- tuning of scales and modes, *major and minor*
- modulation?



Harmony by numbers: applying JI to music

- tuning of the open strings *in perfect fifths*
- tuning of major, minor, and diminished chords
1:3:5 (major); 3:5:15 (minor); 3:5:7 (diminished)
- tuning of scales and modes, *major and minor*
- modulation?



Harmony by numbers: applying JI to music

- tuning of the open strings *in perfect fifths*
- tuning of major, minor, and diminished chords
1:3:5 (major); 3:5:15 (minor); 3:5:7 (diminished)
- tuning of scales and modes, *major and minor*
- modulation!



Harmony by numbers: applying JI to music

- tuning of the open strings *in perfect fifths*
- tuning of major, minor, and diminished chords
1:3:5 (major); 3:5:15 (minor); 3:5:7 (diminished)
- tuning of scales and modes, *major and minor*
- why limit the primes to 7° (and occasionally a 17° as flat 9th)?



Harmony by numbers: applying JI to music repertoire

- tuning of the open strings *in perfect fifths*
- tuning of major, minor, and diminished chords
1:3:5 (major); 3:5:15 (minor); 3:5:7 (diminished)
- tuning of scales and modes, *major and minor*
- why limit the primes to 7° (and occasionally a 17° as flat 9th)?
- why two violins? why not electronics, or more compositional interventions?



Harmony by numbers: applying JI to music repertoire

- tuning of the open strings *in perfect fifths*
- tuning of major, minor, and diminished chords
1:3:5 (major); 3:5:15 (minor); 3:5:7 (diminished)
- tuning of scales and modes, *major and minor*
- why limit the primes to 7° (and occasionally a 17° as flat 9th)?
- why two violins? why not electronics, or more compositional interventions?
- when and how to make choices between pure vertical consonances (introducing more microtonal variation) vs. maintaining melodic simplicity?



Streams barely in winter

three beginnings for Walter

Marc Sabat

1. Cold

Violin 1

Violin 2

3°/IV

[illegible]

attacca

Sonata Prima per Violino Solo senza Basso
Sei Bach-Intonazioni, Ia



Sonata Prima a Violino Solo senza Basso

Sei Bach-Intonazioni per Violino Solo, Ia

Johann Sebastian Bach

intonation and 2nd voice composed by Marc Sabat

Violin (solo)

Adagio

Violin 2 (bordon)

sempre sotto voce

4^o/III 2^o/IV V I

tr

2^b 3

E-natural (not Eb) in MS

tr

4^o/IV V 2^o/IV 2^o/III

Sonata Prima a Violino Solo senza Basso

Sei Bach-Intonazioni per Violino Solo, Ia

Johann Sebastian Bach

Bach begins by showing the notes, in this case a diatonic scale. The pitches are tuned as simple ratios from G and D.

intonation and 2nd voice composed by Marc Sabat

Violin (solo)

Adagio

Violin 2 (bordon)

sempre sotto voce

4^o/III 2^o/IV V I

tr

2b

3

E-natural (not Eb) in MS

tr

4^o/IV V 2^o/IV 2^o/III

Sei Bach-Intonazioni per Violino Solo, la

intonation and 2nd voice composed by Marc Sabat

[illegible]

2b

3

E-natural
(not Eb) in MS

tr

4°/IV

V

2°/IV

2°/III



Streams barely in winter (2019) 1. Cold 2. Sun 3. Stones
three beginnings for Walter played before Bach for two violins

Sei Bach-Intonazioni per Violino Solo (2000–17) Ia IIa IIIa
version in just intonation for violin solo with violin bordun counterpoint

Chords, melodies: a look at harmony by numbers
Sara Cubarsi and Xenia Gogu, violins

Streams barely in winter

three beginnings for Walter

Marc Sabat

1. Cold

Violin 1

Violin 2

The first system of the musical score for '1. Cold' features two staves. Violin 1 (top staff) begins with a half note G4 (F#4 in the key signature), followed by a half note A4, and then a long melodic line starting on B4. Violin 2 (bottom staff) starts with a half note G3, followed by a half note A3, and then a series of eighth notes: B3, A3, G3, F#3, E3, D3, C3, B2, A2, G2, F#2, E2, D2, C2, B1, A1, G1, F#1, E1, D1, C1, B0, A0, G0, F#0, E0, D0, C0, B-1, A-1, G-1, F#-1, E-1, D-1, C-1, B-2, A-2, G-2, F#-2, E-2, D-2, C-2, B-3, A-3, G-3, F#-3, E-3, D-3, C-3, B-4, A-4, G-4, F#-4, E-4, D-4, C-4, B-5, A-5, G-5, F#-5, E-5, D-5, C-5, B-6, A-6, G-6, F#-6, E-6, D-6, C-6, B-7, A-7, G-7, F#-7, E-7, D-7, C-7, B-8, A-8, G-8, F#-8, E-8, D-8, C-8, B-9, A-9, G-9, F#-9, E-9, D-9, C-9, B-10, A-10, G-10, F#-10, E-10, D-10, C-10, B-11, A-11, G-11, F#-11, E-11, D-11, C-11, B-12, A-12, G-12, F#-12, E-12, D-12, C-12, B-13, A-13, G-13, F#-13, E-13, D-13, C-13, B-14, A-14, G-14, F#-14, E-14, D-14, C-14, B-15, A-15, G-15, F#-15, E-15, D-15, C-15, B-16, A-16, G-16, F#-16, E-16, D-16, C-16, B-17, A-17, G-17, F#-17, E-17, D-17, C-17, B-18, A-18, G-18, F#-18, E-18, D-18, C-18, B-19, A-19, G-19, F#-19, E-19, D-19, C-19, B-20, A-20, G-20, F#-20, E-20, D-20, C-20, B-21, A-21, G-21, F#-21, E-21, D-21, C-21, B-22, A-22, G-22, F#-22, E-22, D-22, C-22, B-23, A-23, G-23, F#-23, E-23, D-23, C-23, B-24, A-24, G-24, F#-24, E-24, D-24, C-24, B-25, A-25, G-25, F#-25, E-25, D-25, C-25, B-26, A-26, G-26, F#-26, E-26, D-26, C-26, B-27, A-27, G-27, F#-27, E-27, D-27, C-27, B-28, A-28, G-28, F#-28, E-28, D-28, C-28, B-29, A-29, G-29, F#-29, E-29, D-29, C-29, B-30, A-30, G-30, F#-30, E-30, D-30, C-30, B-31, A-31, G-31, F#-31, E-31, D-31, C-31, B-32, A-32, G-32, F#-32, E-32, D-32, C-32, B-33, A-33, G-33, F#-33, E-33, D-33, C-33, B-34, A-34, G-34, F#-34, E-34, D-34, C-34, B-35, A-35, G-35, F#-35, E-35, D-35, C-35, B-36, A-36, G-36, F#-36, E-36, D-36, C-36, B-37, A-37, G-37, F#-37, E-37, D-37, C-37, B-38, A-38, G-38, F#-38, E-38, D-38, C-38, B-39, A-39, G-39, F#-39, E-39, D-39, C-39, B-40, A-40, G-40, F#-40, E-40, D-40, C-40, B-41, A-41, G-41, F#-41, E-41, D-41, C-41, B-42, A-42, G-42, F#-42, E-42, D-42, C-42, B-43, A-43, G-43, 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G-271, F#-271, E-271, D-271, C-271, B-272, A-272, G-272, F#-272, E-272, D-272, C-272, B-273, A-273, G-273, F#-273, E-273, D-273, C-273, B-274, A-274, G-274, F#-274, E-274, D-274, C-274, B-275, A-275, G-275, F#-275, E-275, D-275, C-275, B-276, A-276, G-276, F#-276, E-276, D-276, C-276, B-277, A-277, G-277, F#-277, E-277, D-277, C-277, B-278, A-278, G-278, F#-278, E-278, D-278, C-278, B-279, A-279, G-279, F#-279, E-279, D-279, C-279, B-280, A-280, G-280, F#-280, E-280, D-280, C-280, B-281, A-281, G-281, F#-281, E-281, D-281, C-281, B-282, A-282, G-282, F#-282, E-282, D-282, C-282, B-283, A-283, G-283, F#-283, E-283, D-283, C-283, B-284, A-284, G-284, F#-284, E-284, D-284, C-284, B-285, A-285, G-285, F#-285, E-285, D-285, C-285, B-286, A-286, G-286, F#-286, E-286, D-286, C-286, B-287, A-287, G-287, F#-287, E-287, D-287, C-287, B-288, A-288, G-288, F#-288, E-288, D-288, C-288, B-289, A-289, G-289, F#-289, E-289, D-289, C-289, B-290, A-290, G-290, F#-290, E-290, D-290, C-290, B-291, A-291, G-291, F#-291, E-291, D-291, C-291, B-292, A-292, G-292, F#-292, E-292, D-292, C-292, B-293, A-293, G-293, F#-293, E-293, D-293, C-293, B-294, A-294, G-294, F#-294, E-294, D-294, C-294, B-295, A-295, G-295, F#-295, E-295, D-295, C-295, B-296, A-296, G-296, F#-296, E-296, D-296, C-296, B-

The first system of the musical score consists of two staves. The upper staff is in treble clef with a key signature of two sharps (F# and C#) and a time signature of 8/8. It begins with a whole note chord, followed by a half note chord, and then a series of notes with slurs and ties. Above the staff, the label '2°/III' is positioned over a note. Below the staff, the labels '3°/IV' and '4°/IV' are positioned over notes. The lower staff is in bass clef and contains a series of notes with slurs and ties, including a whole note chord and a half note chord.

The second system of the musical score consists of two staves. The upper staff is in treble clef and contains a series of notes with slurs and ties, including a whole note chord and a half note chord. Above the staff, the label '3°/III' is positioned over a note. Below the staff, the label '3°/IV' is positioned over a note. The lower staff is in bass clef and contains a series of notes with slurs and ties, including a whole note chord and a half note chord. The system concludes with a double bar line.

attacca
Sonata Prima per Violino Solo senza Basso
Sei Bach-Intonazioni, Ia

Sonata Prima a Violino Solo senza Basso

Sei Bach-Intonazioni per Violino Solo, la

Johann Sebastian Bach

intonation and 2nd voice composed by Marc Sabat

Violin (solo)

Adagio

Violin 2 (bordon)

sempre sotto voce

4°/III 2°/IV

V I

2b

3

E-natural (not Eb) in MS

tr

4°/IV V 2°/IV 2°/III

tr

4

tr

V I 2°/II 2°/IV

2

6

Musical score for measures 6 and 7. Measure 6 features a treble staff with a melodic line and a bass staff with a single note. Measure 7 features a treble staff with a melodic line and a bass staff with a single note. The key signature has one flat (B-flat).

8

Musical score for measures 8 and 9. Measure 8 features a treble staff with a melodic line and a bass staff with a single note. Measure 9 features a treble staff with a melodic line and a bass staff with a single note. The key signature has one flat (B-flat).

9b

10

Musical score for measures 9b and 10. Measure 9b features a treble staff with a melodic line and a bass staff with a single note. Measure 10 features a treble staff with a melodic line and a bass staff with a single note. The key signature has one flat (B-flat).

11

Measures 11 and 12 of a musical score. Measure 11 features a complex melodic line in the right hand with many accidentals and a sustained bass line in the left hand. Measure 12 continues the melodic development with a trill (tr) and a triplet (3) in the right hand. A fermata is placed over the final note of measure 12.

13

Measures 13 and 14 of a musical score. Measure 13 shows a continuation of the melodic line with a trill (tr) and a fermata. Measure 14 features a sustained bass line in the left hand and a melodic line in the right hand. A fermata is placed over the final note of measure 14.

14b

15

Measures 14b and 15 of a musical score. Measure 14b features a melodic line in the right hand with a trill (tr) and a sustained bass line in the left hand. Measure 15 continues the melodic development with a trill (tr) and a sustained bass line in the left hand. A fermata is placed over the final note of measure 15.

4

16

2°/IV

tr

V

V

Detailed description: This system contains measures 16 and 17. Measure 16 features a treble staff with a complex melodic line including sixteenth-note runs and a trill (tr) on the final note. The bass staff has a simple accompaniment with a half note and a quarter note. Measure 17 continues the melodic development in the treble staff, while the bass staff has a half note and a quarter note. A fermata is placed over the final note of the bass staff in measure 17.

18

2°/III

Detailed description: This system contains measures 18 and 19. Measure 18 has a treble staff with a melodic line and a bass staff with a long, sustained note. Measure 19 continues the melodic line in the treble staff, while the bass staff has a half note and a quarter note. A fermata is placed over the final note of the bass staff in measure 19.

20

3°/IV

4°/IV

tr

Detailed description: This system contains measures 20 and 21. Measure 20 features a treble staff with a melodic line and a bass staff with a half note and a quarter note. Measure 21 continues the melodic line in the treble staff, while the bass staff has a half note and a quarter note. A fermata is placed over the final note of the bass staff in measure 21.

Fuga

Allegro

E-natural
(not Eb) in MS

5

5

8

This musical score consists of four systems, each with a treble and bass staff. The key signature has one sharp (F#) and the time signature is 3/4. The notation includes various rhythmic values, accidentals, and dynamic markings.

System 1 (Measures 11-13):

- Measure 11: Treble staff has a half note G4 and a half note A4. Bass staff has a half note G3 and a half note A3. Chordal symbols: $3^{\circ}/IV$ and $4^{\circ}/IV$.
- Measure 12: Treble staff has a half note G4 and a half note A4. Bass staff has a half note G3 and a half note A3. Chordal symbols: $2^{\circ}/III$.
- Measure 13: Treble staff has a half note G4 and a half note A4. Bass staff has a half note G3 and a half note A3. Chordal symbols: $2^{\circ}/III$.

System 2 (Measures 14-17):

- Measure 14: Treble staff has a half note G4 and a half note A4. Bass staff has a half note G3 and a half note A3. Chordal symbols: $2^{\circ}/IV$ and $3^{\circ}/III$.
- Measure 15: Treble staff has a half note G4 and a half note A4. Bass staff has a half note G3 and a half note A3. Chordal symbols: $4^{\circ}/IV$ and $4^{\circ}/III$.
- Measure 16: Treble staff has a half note G4 and a half note A4. Bass staff has a half note G3 and a half note A3. Chordal symbols: $4^{\circ}/II$ and $4^{\circ}/IV$.
- Measure 17: Treble staff has a half note G4 and a half note A4. Bass staff has a half note G3 and a half note A3. Chordal symbols: $4^{\circ}/II$ and $4^{\circ}/IV$.

System 3 (Measures 18-21):

- Measure 18: Treble staff has a half note G4 and a half note A4. Bass staff has a half note G3 and a half note A3. Chordal symbols: $2^{\circ}/III$ and $2^{\circ}/II$.
- Measure 19: Treble staff has a half note G4 and a half note A4. Bass staff has a half note G3 and a half note A3. Chordal symbols: $2^{\circ}/III$ and $2^{\circ}/II$.
- Measure 20: Treble staff has a half note G4 and a half note A4. Bass staff has a half note G3 and a half note A3. Chordal symbols: $2^{\circ}/III$ and $2^{\circ}/II$.
- Measure 21: Treble staff has a half note G4 and a half note A4. Bass staff has a half note G3 and a half note A3. Chordal symbols: $2^{\circ}/III$ and $2^{\circ}/II$.

System 4 (Measures 22-25):

- Measure 22: Treble staff has a half note G4 and a half note A4. Bass staff has a half note G3 and a half note A3. Chordal symbols: $3^{\circ}/II$ and V .
- Measure 23: Treble staff has a half note G4 and a half note A4. Bass staff has a half note G3 and a half note A3. Chordal symbols: $3^{\circ}/II$ and V .
- Measure 24: Treble staff has a half note G4 and a half note A4. Bass staff has a half note G3 and a half note A3. Chordal symbols: $3^{\circ}/II$ and V .
- Measure 25: Treble staff has a half note G4 and a half note A4. Bass staff has a half note G3 and a half note A3. Chordal symbols: $3^{\circ}/II$ and V .

26

7

3°/III 3°/IV 2°/II 4°/IV 2°/II

30

4°/II

34

5°/II 8°/III 4°/I

39

2°/III 2°/II

43

2°/IV

2°/III

46

2°/IV

2°/IV

49

2°/IV

2°/III

52

V

[illegible]

10

69

72

76

79

2°/III

2°/III

2°/III

2°/III

2°/III

4°/III

2°/III

2°/IV

3°/II

4°/III

4°/II

4°/IV

3°/IV

3°/III

2°/IV

83

Measures 83-86 of a musical score. The right hand features a complex melodic line with many accidentals and slurs. The left hand provides a harmonic accompaniment with chords and single notes.

87

Measures 87-89 of a musical score. The right hand continues with a fast, intricate melodic passage. The left hand has a few chords, with a '2°/IV' marking above the first measure.

90

Measures 90-92 of a musical score. The right hand features a fast, flowing melodic line. The left hand has a few chords, with a '2°/IV' marking above the first measure.

93

Measures 93-95 of a musical score. The right hand features a fast, flowing melodic line with a long slur. The left hand has a few chords, with a '2°/IV' marking above the first measure.

12

Siciliana

12

3

5

2°/III

V

V

V

The image displays a musical score for a piece titled "Siciliana". The score is written for piano, featuring a treble and bass staff. The time signature is 8/8. The key signature has one flat (B-flat). The score is divided into three systems, each containing two measures. The first system (measures 12-13) shows a complex melodic line in the treble staff with many beamed sixteenth notes and rests, and a simpler bass line. The second system (measures 14-15) continues the melodic development with a triplet of eighth notes in measure 14 and a long melodic phrase in measure 15. The third system (measures 16-17) features a dense melodic texture in measure 16, including a triplet of eighth notes, and a final measure (17) with a long melodic line. Chordal symbols (V, 2°/III) are placed above the bass staff in measures 13, 15, 16, and 17. The piece concludes with a final note in measure 17.

7 13

Measures 7-13. Treble clef, key signature of one sharp (F#). Measure 7 starts with a treble clef and a key signature of one sharp. The melody features eighth and sixteenth notes with various accidentals. The bass line has a whole note and a half note. Measure 8 continues the melody with a trill and a grace note. Measure 9 has a trill and a grace note. Measure 10 has a trill and a grace note. Measure 11 has a trill and a grace note. Measure 12 has a trill and a grace note. Measure 13 has a trill and a grace note.

9

3rd/IV

4th/IV

Measures 9-11. Treble clef, key signature of one sharp (F#). Measure 9 starts with a treble clef and a key signature of one sharp. The melody features eighth and sixteenth notes with various accidentals. The bass line has a whole note and a half note. Measure 10 continues the melody with a trill and a grace note. Measure 11 has a trill and a grace note.

11

Measures 11-13. Treble clef, key signature of one sharp (F#). Measure 11 starts with a treble clef and a key signature of one sharp. The melody features eighth and sixteenth notes with various accidentals. The bass line has a whole note and a half note. Measure 12 continues the melody with a trill and a grace note. Measure 13 has a trill and a grace note.

13

Measures 13-15. Treble clef, key signature of one sharp (F#). Measure 13 starts with a treble clef and a key signature of one sharp. The melody features eighth and sixteenth notes with various accidentals. The bass line has a whole note and a half note. Measure 14 continues the melody with a trill and a grace note. Measure 15 has a trill and a grace note.

14

15

Two staves of music. The top staff (treble clef) contains measures 14 and 15. Measure 14 has a complex melody with many accidentals and slurs. Measure 15 continues the melody. The bottom staff (treble clef) contains measures 14 and 15. Measure 14 has a simple accompaniment with a few notes and rests. Measure 15 continues the accompaniment. A 'V' mark is placed above the first note of the bottom staff in measure 15.

17

Two staves of music. The top staff (treble clef) contains measures 16 and 17. Measure 16 has a complex melody with many accidentals and slurs. Measure 17 continues the melody. The bottom staff (treble clef) contains measures 16 and 17. Measure 16 has a simple accompaniment with a few notes and rests. Measure 17 continues the accompaniment. 'V' marks are placed above the first notes of the bottom staff in both measures 16 and 17.

19

Two staves of music. The top staff (treble clef) contains measures 18 and 19. Measure 18 has a complex melody with many accidentals and slurs. Measure 19 continues the melody. The bottom staff (treble clef) contains measures 18 and 19. Measure 18 has a simple accompaniment with a few notes and rests. Measure 19 continues the accompaniment. A 'V' mark is placed above the first note of the bottom staff in measure 18.

Presto

Measures 1-7 of a musical score in 3/8 time, marked *Presto*. The key signature has two flats (B-flat and E-flat). The music features rapid sixteenth-note passages in both hands, with some triplets and slurs.

8

Measures 8-14 of the musical score. Measure 8 is marked with a large '8'. The music continues with rapid sixteenth-note passages and slurs.

15

Measures 15-21 of the musical score. Measure 15 is marked with a large '15'. The music continues with rapid sixteenth-note passages and slurs.

22

Measures 22-28 of the musical score. Measure 22 is marked with a large '22'. The music continues with rapid sixteenth-note passages and slurs.

16

29

System 1 (measures 16-22): The right hand features a complex melodic line with many accidentals (sharps, flats, naturals) and slurs. The left hand provides a harmonic accompaniment with chords and single notes.

36

System 2 (measures 23-30): The right hand continues with intricate melodic patterns. The left hand has long horizontal lines with occasional notes, suggesting sustained chords or a specific rhythmic pattern.

44

System 3 (measures 31-38): The right hand shows a series of descending and ascending melodic phrases. The left hand includes a measure with the instruction $2^{\circ}/\Pi$ and a fermata, indicating a second ending or a specific performance instruction.

52

System 4 (measures 39-46): The right hand features a double bar line followed by a repeat sign, indicating a repeated melodic figure. The left hand continues with its accompaniment, including some chords and moving lines.

Presto

Measures 1-7 of a musical score in 3/8 time, marked Presto. The key signature has two flats (B-flat and E-flat). The music features rapid sixteenth-note passages in both hands, with some slurs and ties. The right hand has a few sharp accidentals in measures 5 and 6.

8

Measures 8-14 of the musical score. Measure 8 is marked with a large '8'. The music continues with rapid sixteenth-note patterns. Measure 11 has a sharp accidental on the G note in the right hand. Measure 14 has a slur over the final two notes of the right hand.

15

Measures 15-21 of the musical score. Measure 15 is marked with a large '15'. The music continues with rapid sixteenth-note patterns. Measure 18 has a sharp accidental on the G note in the right hand. Measure 21 has a slur over the final two notes of the right hand.

22

Measures 22-28 of the musical score. Measure 22 is marked with a large '22'. The music continues with rapid sixteenth-note patterns. Measure 25 has a sharp accidental on the G note in the right hand. Measure 28 has a slur over the final two notes of the right hand.

16

29

System 1 (measures 16-22): The right hand features a complex melodic line with many accidentals (sharps, flats, naturals) and slurs. The left hand provides a harmonic accompaniment with chords and single notes.

36

System 2 (measures 23-30): The right hand continues with intricate melodic patterns. The left hand has long horizontal lines with occasional notes, suggesting sustained chords or a specific rhythmic pattern.

44

System 3 (measures 31-38): The right hand shows a series of descending and ascending melodic phrases. The left hand includes a measure with the instruction $2^{\circ}/\Pi$ and a fermata, indicating a second ending or a specific performance instruction.

52

System 4 (measures 39-46): The right hand features a double bar line followed by a repeat sign, indicating a repeated melodic figure. The left hand continues with its accompaniment, including some slurs and accidentals.

59

Measures 59-65 of a musical score. The system consists of two staves. The upper staff features a complex melodic line with many beamed sixteenth and thirty-second notes, including trills and grace notes. The lower staff provides a harmonic accompaniment with a mix of eighth and sixteenth notes, some beamed together. The key signature has one flat (B-flat), and the time signature is 4/4.

66

Measures 66-72 of a musical score. The system consists of two staves. The upper staff continues the intricate melodic pattern with frequent beaming and trills. The lower staff's accompaniment includes some longer note values and rests. The key signature remains one flat (B-flat), and the time signature is 4/4.

73

Measures 73-79 of a musical score. The system consists of two staves. The upper staff shows a continuation of the fast, melodic line. The lower staff features a more active accompaniment with many beamed sixteenth notes. The key signature remains one flat (B-flat), and the time signature is 4/4.

80

Measures 80-86 of a musical score. The system consists of two staves. The upper staff continues the melodic development with various ornaments and beamed notes. The lower staff provides a steady accompaniment. The key signature remains one flat (B-flat), and the time signature is 4/4.

18

87

Measures 87-93 of a musical score. The system consists of two staves. The upper staff contains a complex melodic line with many accidentals (sharps and flats) and slurs. The lower staff contains a simpler accompaniment with mostly whole and half notes, some with accidentals.

94

Measures 94-100 of a musical score. The system consists of two staves. The upper staff continues the complex melodic line. The lower staff features more rhythmic activity, including eighth and sixteenth notes, and rests.

101

Measures 101-107 of a musical score. The system consists of two staves. The upper staff has a melodic line with slurs. The lower staff has a bass line with some long notes and rests.

108

Measures 108-114 of a musical score. The system consists of two staves. The upper staff continues the melodic line. The lower staff includes figured bass notation: $3^{\circ}/IV$, $4^{\circ}/IV$, and $3^{\circ}/III$ above the notes.

115

Musical score for measures 115-121. The system consists of two staves. The upper staff is in treble clef with a key signature of one flat (B-flat). It contains a series of eighth and sixteenth notes, often beamed together in groups of four or six. The lower staff is in treble clef and contains mostly whole and half notes, some with long horizontal lines indicating sustained sounds or glissandi.

122

Musical score for measures 122-128. The system consists of two staves. The upper staff continues with complex rhythmic patterns of eighth and sixteenth notes. The lower staff features whole and half notes, with some measures containing long horizontal lines.

129

2°/III

Musical score for measures 129-135. The system consists of two staves. The upper staff continues with complex rhythmic patterns. The lower staff features whole and half notes, with some measures containing long horizontal lines. The system concludes with a double bar line and repeat dots.

16

29

System 1 (measures 16-22) features a treble and bass staff. The treble staff contains a complex melodic line with many accidentals (sharps, flats, naturals) and slurs. The bass staff provides a harmonic accompaniment with fewer notes and some accidentals.

36

System 2 (measures 23-30) continues the melodic and harmonic development. The treble staff has dense, fast-moving passages, while the bass staff has more sustained notes with some slurs.

44

System 3 (measures 31-38) includes a key signature change to two flats (B-flat and E-flat) in measure 34, indicated by a double bar line and the notation $2^{\circ}/\text{II}$. The treble staff continues with intricate melodic patterns, and the bass staff has a more active line with some slurs.

52

System 4 (measures 39-46) features a repeat sign in measure 40. The treble staff has a melodic line with many accidentals, and the bass staff has a more active line with some slurs.

59

Measures 59-65 of a musical score. The system consists of two staves. The upper staff features a complex melodic line with many beamed sixteenth and thirty-second notes, including trills and grace notes. The lower staff provides a harmonic accompaniment with a mix of eighth and sixteenth notes, some beamed together. The key signature has one flat (B-flat), and the time signature is 4/4.

66

Measures 66-72 of a musical score. The system consists of two staves. The upper staff continues the intricate melodic pattern with frequent beaming and trills. The lower staff maintains a steady accompaniment. The key signature remains one flat (B-flat), and the time signature is 4/4.

73

Measures 73-79 of a musical score. The system consists of two staves. The upper staff shows a continuation of the fast, melodic line. The lower staff's accompaniment includes some longer note values and rests. The key signature remains one flat (B-flat), and the time signature is 4/4.

80

Measures 80-86 of a musical score. The system consists of two staves. The upper staff features a melodic line with many beamed notes and trills. The lower staff provides a rhythmic accompaniment. The key signature remains one flat (B-flat), and the time signature is 4/4.

18

87

System 1 (measures 87-93). The right hand features a complex melodic line with many accidentals and slurs. The left hand plays a simple bass line with dotted half notes.

94

System 2 (measures 94-100). The right hand continues with a complex melodic line. The left hand has a more active bass line with eighth notes and rests.

101

System 3 (measures 101-107). The right hand has a complex melodic line with many slurs. The left hand has a complex bass line with many slurs and ties.

108

System 4 (measures 108-114). The right hand continues with a complex melodic line. The left hand has a complex bass line with many slurs and ties. Chord symbols $3^\circ/IV$, $4^\circ/IV$, and $3^\circ/III$ are present below the bass line.

115

Musical score for measures 115-121. The system consists of two staves. The upper staff is in treble clef with a key signature of one flat (B-flat). It contains a series of eighth and sixteenth notes, often beamed together in groups of four or six. The lower staff is in treble clef and contains mostly half notes and whole notes, some with long horizontal lines indicating sustained sounds or glissandi.

122

Musical score for measures 122-128. The system consists of two staves. The upper staff continues with complex rhythmic patterns in treble clef. The lower staff features half notes and whole notes, with some measures containing long horizontal lines.

129

2^o/III

Musical score for measures 129-135. The system consists of two staves. The upper staff continues with complex rhythmic patterns in treble clef. The lower staff features half notes and whole notes, with some measures containing long horizontal lines. The system concludes with a double bar line and repeat dots.