



Marc Sabat

www.plainsound.org

masa@plainsound.org

**DocMus Applied Study Scientific / Artistic Research
Sibelius Academy / Uniarts Helsinki**



Marc Sabat

www.plainsound.org

masa@plainsound.org

DocMus Applied Study Scientific / Artistic Research

First Doctoral Presentation, 19.10.24, Organo



Marc Sabat

www.plainsound.org

masa@plainsound.org

Chords, melodies: a look at harmony by numbers



Marc Sabat

www.plainsound.org

masa@plainsound.org

Chords, melodies: a look at harmony by numbers

Tuning Bach – an experimental recomposition



Marc Sabat

www.plainsound.org

masa@plainsound.org

Chords, melodies: a look at harmony by numbers

Sara Cubarsi and Xenia Gogu, violins



Streams barely in winter (2019) 1. *Cold* 2. *Sun* 3. *Stones*
three beginnings for Walter played before Bach for two violins

Sei Bach-Intonazioni per Violino Solo (2000–17) Ia IIa IIIa
version in just intonation for violin solo with violin bordun counterpoint

Chords, melodies: a look at harmony by numbers
Sara Cubarsi and Xenia Gogu, violins



Harmony by numbers

- harmony ?



Harmony by numbers

- harmony ?
- numbers ?



Harmony by numbers | “*armonia*” = fitting together

- harmony
- numbers ?



Harmony by numbers | “*armonia*” = fitting together

- harmony = relations between pitches other than higher / lower (Tenney)
- numbers ?



Harmony by numbers | “*overtones*” = aliquot divisions

- harmony = relations between pitches other than higher / lower
- numbers



Harmony by numbers | “*overtones*” = aliquot divisions

- harmony = relations between pitches other than higher / lower
- numbers = harmonic partial row (Aristotle, Mersenne, Sauveur, Helmholtz, et al.)



Harmony by numbers | “*overtones*” = aliquot divisions

- harmony = relations between pitches other than higher / lower
- numbers = harmonic partial row; ratios of partials = rational intervals (Partch et al.)



Harmony by numbers = relations of harmonic partials

- harmony = relations between pitches other than higher / lower
- numbers = harmonic partial row; ratios of partials = rational intervals



Harmony by numbers = rational intonation (JI)

- harmony = relations between pitches other than higher / lower
- numbers = harmonic partial row; ratios of partials = rational intervals



Harmony by numbers: research questions

- What is rational intonation or JI?



Harmony by numbers: research questions

- What is rational intonation or JI? *a practice of intonation by rational intervals*



Harmony by numbers: research questions

- What is rational intonation or JI? *a practice of intonation by rational intervals*
- What does it sound like?



Harmony by numbers: research questions

- What is rational intonation or JI? *a practice of intonation by rational intervals*
- What does it sound like? *intervals, chords, timbres, melodies ...*



Harmony by numbers: research questions

- What is rational intonation or JI? *a practice of intonation by rational intervals*
- What does it sound like? *intervals, chords, timbres, melodies ... tools?*



Harmony by numbers: research questions

- What is rational intonation or JI? *a practice of intonation by rational intervals*
- What does it sound like? *intervals, chords, timbres, melodies ... tools? ... notation*



Harmony by numbers: research questions

- What is rational intonation or JI? *a practice of intonation by rational intervals*
- What does it sound like? *intervals, chords, timbres, melodies ... tools? ... notation*
- Repertoire?



Harmony by numbers: research questions

- What is rational intonation or JI? *a practice of intonation by rational intervals*
- What does it sound like? *intervals, chords, timbres, melodies ... tools? ... notation*
- Repertoire: how does one make decisions about tuning an existing piece in JI?



Harmony by numbers: research questions

- What is rational intonation or JI? *a practice of intonation by rational intervals*
- What does it sound like? *intervals, chords, timbres, melodies ... tools? ... notation*
- Repertoire: how does one make decisions about tuning an existing piece in JI?
making choices that prioritise tuneable, consonant intervals sometimes requires microtonal pitch variations based on context (i.e., microtonal modulations)



Harmony by numbers: research questions

- What is rational intonation or JI? *a practice of intonation by rational intervals*
- What does it sound like? *intervals, chords, timbres, melodies ... tools? ... notation*
- Repertoire: how does one make decisions about tuning an existing piece in JI?
making choices that prioritise tuneable, consonant intervals sometimes requires microtonal pitch variations based on context (i.e., microtonal modulations)
- How can a performer learn to hear and play such tunings accurately?



Harmony by numbers: research questions

- What is rational intonation or JI? *a practice of intonation by rational intervals*
- What does it sound like? *intervals, chords, timbres, melodies ... tools? ... notation*
- Repertoire: how does one make decisions about tuning an existing piece in JI?
making choices that prioritise tuneable, consonant intervals sometimes requires microtonal pitch variations based on context (i.e., microtonal modulations)
- How can a performer learn to hear and play such tunings accurately?
- How does JI relate to historically aware performance practices of earlier music?



Harmony by numbers: research questions

- What is rational intonation or JI? *a practice of intonation by rational intervals*
- What does it sound like? *intervals, chords, timbres, melodies ... tools? ... notation*
- Repertoire: how does one make decisions about tuning an existing piece in JI?
making choices that prioritise tuneable, consonant intervals sometimes requires microtonal pitch variations based on context (i.e., microtonal modulations)
- How can a performer learn to hear and play such tunings accurately?
- How does JI relate to historically aware performance practices of earlier music?
pythagorean, meantone, well-tempered tuning systems



Harmony by numbers: research questions

- What is rational intonation or JI? *a practice of intonation by rational intervals*
- What does it sound like? *intervals, chords, timbres, melodies ... tools? ... notation*
- Repertoire: how does one make decisions about tuning an existing piece in JI?
making choices that prioritise tuneable, consonant intervals sometimes requires microtonal pitch variations based on context (i.e., microtonal modulations)
- How can a performer learn to hear and play such tunings accurately?
- How does JI relate to historically aware performance practices of earlier music?
pythagorean, meantone, well-tempered tuning systems
- What new forms of music are emerging from JI practices? How can knowledge about intonation shape the creation of new music?

Case Study 1: Tuning Bach

Tuning Bach features an experimental recomposition of J.S. Bach's three solo violin sonatas: an adaptation for two violins in rational intonation by Marc Sabat.



Case Study 1: Tuning Bach

Tuning Bach features an experimental recomposition of J.S. Bach's three solo violin sonatas: an adaptation for two violins in rational intonation by Marc Sabat.

- project began alongside the development of HEJI notation, in the early 2000's



Case Study 1: Tuning Bach

Tuning Bach features an experimental recomposition of J.S. Bach's three solo violin sonatas, an adaptation for two violins in rational intonation by Marc Sabat.

- project began alongside the development of HEJI notation, in the early 2000's
- basic questions: is it possible to tune Bach's solo violin works by strictly following the logic of rational intonation, using intervals tuneable by ear?



Case Study 1: Tuning Bach

Tuning Bach features an experimental recomposition of J.S. Bach's three solo violin sonatas, an adaptation for two violins in rational intonation by Marc Sabat.

- project began alongside the development of HEJI notation, in the early 2000's
- basic questions: is it possible to tune Bach's solo violin works by strictly following the logic of rational intonation, using intervals tuneable by ear? how might it sound?



Case Study 1: Tuning Bach

Tuning Bach features an experimental recomposition of J.S. Bach's three solo violin sonatas, an adaptation for two violins in rational intonation by Marc Sabat.

- project began alongside the development of HEJI notation, in the early 2000's
- basic questions: is it possible to tune Bach's solo violin works by strictly following the logic of rational intonation, using intervals tuneable by ear? how might it sound? can it be played?



Case Study 1: Tuning Bach

Tuning Bach features an experimental recomposition of J.S. Bach's three solo violin sonatas, an adaptation for two violins in rational intonation by Marc Sabat.

- project began alongside the development of HEJI notation, in the early 2000's
- basic questions: is it possible to tune Bach's solo violin works by strictly following the logic of rational intonation, using intervals tuneable by ear? how might it sound? can it be played? would the interpretation be musically convincing (to me)?



Case Study 1: Tuning Bach

Tuning Bach features an experimental recomposition of J.S. Bach's three solo violin sonatas, an adaptation for two violins in rational intonation by Marc Sabat.

- project began alongside the development of HEJI notation, in the early 2000's
- basic questions: is it possible to tune Bach's solo violin works by strictly following the logic of rational intonation, using intervals tuneable by ear? how might it sound? can it be played? would the interpretation be musically convincing (to me)?

In the mid 2010's, I began composing a second violin part, with the intention of providing sustaining tones that allow JI intervals to be tuned more precisely by ear; gradually this evolved into a contrapuntal conversation with Bach's music.



Case Study 1: Streams barely in winter

Tuning Bach features an experimental recomposition of J.S. Bach's three solo violin sonatas, an adaptation for two violins in rational intonation by Marc Sabat.

In 2019, after making the Bach intonations, three short preludes were composed, one for each sonata. These miniatures for two violins take emblematic JI elements of the respective Bach sonatas as points of departure, introducing the sounds of various kinds of JI intervals.

Case Study 1: recording and live concert

Tuning Bach features an experimental recomposition of J.S. Bach's three solo violin sonatas, an adaptation for two violins in rational intonation by Marc Sabat.

In 2019, after making the Bach intonations, three short preludes were composed, one for each sonata. These miniatures for two violins take emblematic JI elements of the respective Bach sonatas as points of departure, introducing the sounds of various kinds of JI intervals.

Both the preludes and the adaptations were recorded in a studio setting and edited for upcoming release. This lecture-concert, presented as part of my doctoral research, marks the first live performance of the complete project.



Harmony by numbers: a brief tour of JI Fundamentals

- the harmonic partial row



Harmony by numbers: a brief tour of JI Fundamentals

- the harmonic partial row
- HEJI Notation



Harmony by numbers: a brief tour of JI Fundamentals

- the harmonic partial row
- HEJI Notation
- JI intervals and chords



Harmony by numbers: a brief tour of JI Fundamentals

- the harmonic partial row
- HEJI Notation
- JI intervals and chords
- listening techniques for tuning JI sounds



Harmony by numbers: harmonic partials

form a sequence of frequencies which are whole-number multiples of a generating pitch, called the fundamental.



Harmony by numbers: harmonic partials

form a sequence of frequencies which are whole-number multiples of a generating pitch, called the fundamental. Each new harmonic partial fits into the vibrating period of the fundamental, creating a fused harmonic sound. As partials are added, the colour or *timbre* of the sound is enriched or brightened. As lower partials disappear, the sense of fundamental (or periodicity pitch) lingers, creating a *virtual fundamental* enhanced by *combination* or *summation and difference tones*. These *psychoacoustic* phenomena form the basis for how combinations of pitches, even the slightly detuned ones found in the equal tempered system, form harmonies (intervals, chords, aggregates, scales and modes). The concept of *just* or *rational intonation* (JI) is based on the untempered tunings found in the harmonic partial row. *Extended JI* refers to harmonies including the microtonal higher prime partials, 7°, 11°, 13°, etc.

Harmony by numbers: notating JI (HEJI)

Every partial that is a multiple of only 2's and 3's is part of a sequence of perfect fifths and fourths, commonly called "Pythagorean" tuning (with a history dating back to Ancient Mesopotamia). It is written with "normal" flats, naturals, or sharps.



Harmony by numbers: odd partials = new pitch-classes

Every partial that is a multiple of only 2's and 3's is part of a sequence of perfect fifths and fourths, commonly called "Pythagorean" tuning (with a history dating back to Ancient Mesopotamia). It is written with "normal" flats, naturals, or sharps.

Multiplying partials by 2 transposes them up an octave. Therefore, *even-numbered* partials are octave transpositions of pitches occurring previously, while *odd-numbered* partials represent the first (and lowest) occurrences of new pitch classes.



Harmony by numbers: prime partials = new accidentals

Every partial that is a multiple of only 2's and 3's is part of a sequence of perfect fifths and fourths, commonly called "Pythagorean" tuning (with a history dating back to Ancient Mesopotamia). It is written with "normal" flats, naturals, or sharps.

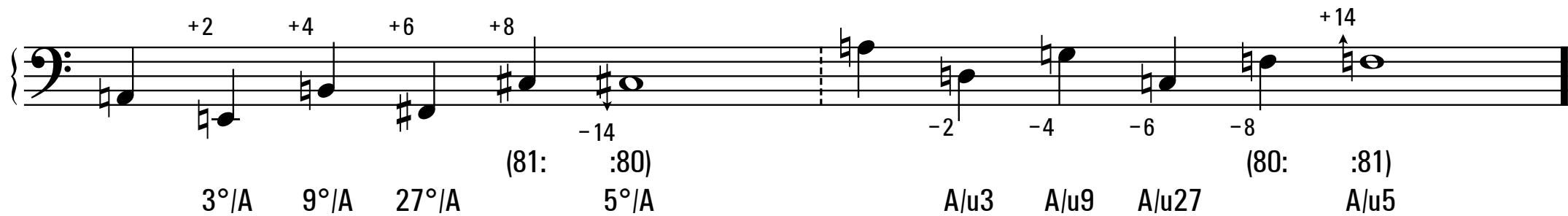
Odd partials that are *prime* numbers always need an additional *modifying* accidental to notate microtonal deviation from a nearby "normal" note. The inverted version of an accidental shows the same deviation occurring *below* a reference pitch.

Harmony by numbers: the Syntonic Comma

Every partial that is a multiple of only 2's and 3's is part of a sequence of perfect fifths and fourths, commonly called "Pythagorean" tuning (with a history dating back to Ancient Mesopotamia). It is written with "normal" flats, naturals, or sharps.

Odd partials that are *prime* numbers always need an additional *modifying* accidental to notate microtonal deviation from a nearby “normal” note. The inverted version of an accidental shows the same deviation occurring *below* a reference pitch.

Notation of 5° partial and its inversion u5 as deviations of a Syntonic Comma from the nearby Pythagorean notes





The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes
with dedicated microtonal accidental symbols for primes 5 through 47

revised by Marc Sabat & Thomas Nicholson

in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above ♯A

8^o partial interval alteration (81:80)

5° M3 (64:63)

7° m7 (32:33)

11° P4 (32:33)

13° M6 (27:26)

17° aug8 (2187:2176)

19° m3 (512:513)

23° aug4 (729:736)

29° m7 (256:261)

31° P8 (32:31)

37° M2 (36:37)

41° M3 (81:82)

43° P4 (128:129)

47° aug4 (729:752)

Standard utonal notation below ♯E

u5 M3 (80:81)

u7 m7 (63:64)

u11 P4 (33:32)

u13 M6 (26:27)

u17 aug8 (2176:2187)

u19 m3 (513:512)

u23 aug4 (736:729)

u29 m7 (261:256)

u31 P8 (31:32)

u37 M2 (37:36)

u41 M3 (82:81)

u43 P4 (129:128)

u47 aug4 (752:729)



The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes
with dedicated microtonal accidental symbols for primes 5 through 47

revised by Marc Sabat & Thomas Nicholson

in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above \natural A

Standard otonal notation above \natural A

8 -14 partial interval alteration (81:80)

+2 -14 +51 +2 +5 +4 +51 +28 +2

7° m7 P4 M6 13° 17° 19° 23°

(64:63) (32:33) (27:26) (2187:2176) (512:513) (729:736)

+6 -27 +30 +5 +4 +51 +29 +12 +51 +28 E-34 +2 E+38

29° m7 P8 31° M2 41° M3 P4 43° 47°

(256:261) (32:31) (36:37) (81:82) (128:129) (729:752)

Standard utonal notation below \natural E

+2 8 -14 +2 +16 +33 +2 +16 +2 +4 +16 +31

u5 M3 u7 m7 P4 u11 u13 u17 u19 u23

(80:81) (63:64) (33:32) (26:27) (2176:2187) (513:512) (736:729)

+29 G-39 +33 +14 F-43 +2 +47 +4 +16 +31 +12 +12 +26

u29 m7 P8 M2 M3 P4 aug4

(261:256) (31:32) (37:36) (82:81) (129:128) (752:729)



The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes
with dedicated microtonal accidental symbols for primes 5 through 47

revised by Marc Sabat & Thomas Nicholson

in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above ♯A

8^o partial interval alteration (81:80)

5° M3 (64:63)

7° m7 (32:33)

11° P4 (32:33)

13° M6 (27:26)

17° aug8 (2187:2176)

19° m3 (512:513)

23° aug4 (729:736)

29° m7 (256:261)

31° P8 (32:31)

37° M2 (36:37)

41° M3 (81:82)

43° P4 (128:129)

47° aug4 (729:752)

Standard utonal notation below ♯E

u5 M3 (80:81)

u7 m7 (63:64)

u11 P4 (33:32)

u13 M6 (26:27)

u17 aug8 (2176:2187)

u19 m3 (513:512)

u23 aug4 (736:729)

u29 m7 (261:256)

u31 P8 (31:32)

u37 M2 (37:36)

u41 M3 (82:81)

u43 P4 (129:128)

u47 aug4 (752:729)



The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes
with dedicated microtonal accidental symbols for primes 5 through 47

revised by Marc Sabat & Thomas Nicholson

in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above ♯A

Notes above ♯A:

- 8^u (81:80) partial interval alteration
- 5° (64:63) M3
- 7° (32:33) m7
- 11° (32:33) P4
- 13° (27:26) M6
- 17° (2187:2176) aug8
- 19° (512:513) m3
- 23° (729:736) aug4
- 29° (256:261) m7
- 31° (32:31) P8
- 37° (36:37) M2
- 41° (81:82) M3
- 43° (128:129) P4
- 47° (729:752) aug4

Standard utonal notation below ♯E

Notes below ♯E:

- u5 (80:81) M3
- u7 (63:64) m7
- u11 (33:32) P4
- u13 (26:27) M6
- u17 (2176:2187) aug8
- u19 (513:512) m3
- u23 (736:729) aug4
- u29 (261:256) m7
- u31 (31:32) P8
- u37 (37:36) M2
- u41 (82:81) M3
- u43 (129:128) P4
- u47 (752:729) aug4



The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes
with dedicated microtonal accidental symbols for primes 5 through 47

revised by Marc Sabat & Thomas Nicholson

in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above ♯A

8^o partial interval alteration (81:80)

5° M3 (64:63)

7° m7 (32:33)

11° P4 (32:33)

13° M6 (27:26)

17° aug8 (2187:2176)

19° m3 (512:513)

23° aug4 (729:736)

29° m7 (256:261)

31° P8 (32:31)

37° M2 (36:37)

41° M3 (81:82)

43° P4 (128:129)

47° aug4 (729:752)

Standard utonal notation below ♯E

u5 M3 (80:81)

u7 m7 (63:64)

u11 P4 (33:32)

u13 M6 (26:27)

u17 aug8 (2176:2187)

u19 m3 (513:512)

u23 aug4 (736:729)

u29 m7 (261:256)

u31 P8 (31:32)

u37 M2 (37:36)

u41 M3 (82:81)

u43 P4 (129:128)

A+36 u47 aug4 (752:729)



The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes
with dedicated microtonal accidental symbols for primes 5 through 47

Notes with “normal” accidentals are tuned in pure fifths

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

revised by Marc Sabat & Thomas Nicholson

in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz

Standard otonal notation above $\natural A$

Standard utonal notation below $\natural E$



The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes
with dedicated microtonal accidental symbols for primes 5 through 47

partial 5° and its multiples are indicated by an arrow

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

revised by Marc Sabat & Thomas Nicholson

in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz

Standard otonal notation above $\natural A$

partial interval alteration 5° M3 (81:80) 7° m7 (64:63) 11° P4 (32:33) 13° M6 (27:26) 17° aug8 (2187:2176) 19° m3 (512:513) 23° aug4 (729:736)

29° m7 (256:261) 31° P8 (32:31) 37° M2 (36:37) 41° M3 (81:82) 43° P4 (128:129) 47° aug4 (729:752)

Standard utonal notation below $\natural E$

in the subharmonic row, the same intervals appear, but are *inverted*

partial interval alteration 5° M3 (80:81) $u5$ M3 (80:81) $u7$ m7 (63:64) $u11$ P4 (33:32) $u13$ M6 (26:27) $u17$ aug8 (2176:2187) $u19$ m3 (513:512) $u23$ aug4 (736:729)

$u29$ m7 (261:256) $u31$ P8 (31:32) $u37$ M2 (37:36) $u41$ M3 (82:81) $u43$ P4 (129:128) $u47$ aug4 (752:729)



The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

partial 7° and its multiples (septimal notes) are indicated by Tartini's hook

revised by Marc Sabat & Thomas Nicholson

in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above ♯A

8^o partial interval alteration (81:80) 5° m7 (64:63) 11° P4 (32:33) 13° M6 (27:26) 17° aug8 (2187:2176) 19° m3 (512:513) 23° aug4 (729:736) 29° m7 (256:261) 31° P8 (32:31) 37° M2 (36:37) 41° M3 (81:82) 43° P4 (128:129) 47° aug4 (729:752) E+38

Standard utonal notation below ♯E

+2 8^o +2 u5 M3 (80:81) u7 m7 (63:64) +33 +2 +16 +16 G#-39 +33 +14 +2 +4 +16 +31 -2 -49 u11 P4 (33:32) u13 M6 (26:27) u17 aug8 (2176:2187) u19 m3 (513:512) u23 aug4 (736:729) +29 G#-39 +33 +14 F-43 +2 +47 +47 +2 +4 C#-41 +16 +31 +12 +4 +16 +31 -4 -28 -51 -3 -2 -49 -27 -10 -49 -26 A+36 +47 A-36 u29 m7 (261:256) u31 P8 (31:32) u37 M2 (37:36) u41 M3 (82:81) u43 P4 (129:128) aug4 (752:729)



The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

partial 9° is odd (new pitch-class) but it isn't prime...

revised by Marc Sabat & Thomas Nicholson

in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above ♯A

Standard utonal notation below ♯E



The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

undecimal notes:

partial 11° and its multiples are indicated by a 2-stroke cross

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes
with dedicated microtonal accidental symbols for primes 5 through 47

revised by Marc Sabat & Thomas Nicholson

in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above $\natural A$

Standard utonal notation below $\natural E$

... or a reversed flat



The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

tridecimal notes:

partial 13° and its multiples are indicated by a 3-stroke reverse flat

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes
with dedicated microtonal accidental symbols for primes 5 through 47

revised by Marc Sabat & Thomas Nicholson

in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above $\natural A$

8^o
partial interval alteration (81:80)

5^o
M3 (64:63)

7^o
m7 (32:33)

11^o
P4 (32:33)

13^o
M6 (27:26)

17^o
aug8 (2187:2176)

19^o
m3 (512:513)

23^o
aug4 (729:736)

29^o
m7 (256:261)

31^o
P8 (32:31)

37^o
M2 (36:37)

41^o
M3 (81:82)

43^o
P4 (128:129)

47^o
aug4 (729:752)

Standard utonal notation below $\natural E$

$\natural 5$
M3 (80:81)

$\natural 7$
m7 (63:64)

$\natural 11$
P4 (33:32)

$\natural 13$
M6 (26:27)

$\natural 17$
aug8 (2176:2187)

$\natural 19$
m3 (513:512)

$\natural 23$
aug4 (736:729)

$\natural 29$
G \sharp -39 (261:256)

$\natural 31$
P8 (31:32)

$\natural 37$
M2 (37:36)

$\natural 41$
M3 (82:81)

$\natural 43$
P4 (129:128)

$\natural 47$
aug4 (752:729)



The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes with dedicated microtonal accidental symbols for primes 5 through 47

revised by Marc Sabat & Thomas Nicholson

*in collaboration with Wolfgang von Schueinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schueinitz*

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol, cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above $\natural A$

as the row approaches 16° , the steps become semitones

partial interval alteration (81:80) (64:63) (32:33) (27:26) (2187:2176) (512:513) (729:736)

29° m7 (256:261) 31° P8 (32:31) 37° M2 (36:37) 41° M3 (81:82) 43° P4 (128:129) 47° aug4 (729:752)

Standard utoctal notation below



The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

The harmonic partial row is a source for intervals used in music:

revised by Marc Sabat & Thomas Nicholson

in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above ♯A

8 partial interval alteration (81:80) 5° (64:63) 7° (32:33) 11° (27:26) 13° (F+41) 17° (2187:2176) 19° (512:513) 23° (729:736)

-27 F+41 -31 29° (256:261) m7 (32:31) 31° P8 (36:37) 37° M2 (81:82) 41° (128:129) M3 43° P4 (729:752) 47° aug4

Standard utonal notation below ♯E

+29 G#-39 +33 +14 F-43 +2 -4 -28 u5 M3 (80:81) u7 m7 (63:64) +16 -2 -49 u11 P4 (33:32) u13 M6 (26:27) +4 C#-41 +16 +31 +2 -3 -2 -49 u17 aug8 (2176:2187) u19 m3 (513:512) +12 +4 -31 -2 -49 u23 aug4 (736:729)

-4 -28 -51 -3 -2 -49 -27 -10 -49 -26 -47 -36 -47 -36 -47 -36 u29 m7 (261:256) u31 P8 (31:32) u37 M2 (37:36) u41 M3 (82:81) u43 P4 (129:128) aug4 (752:729)

The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

The harmonic partial row is a source for intervals used in music:

revised by Marc Sabat & Thomas Nicholson

in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above $\natural A$

an Octave with harmonic ratio 1:2

Standard otonal notation above $\natural A$

an Octave with harmonic ratio 1:2

8^o partial interval alteration (81:80) 5^o M3 (64:63) 7^o m7 (32:33) 11^o P4 (32:33) 13^o M6 (27:26) 17^o aug8 (2187:2176) 19^o m3 (512:513) 23^o aug4 (729:736)

29^o m7 (256:261) 31^o P8 (32:31) 37^o M2 (36:37) 41^o M3 (81:82) 43^o P4 (128:129) 47^o aug4 (729:752)

Standard utonal notation below $\natural E$

Standard utonal notation below $\natural E$

u5 M3 (80:81) u7 m7 (63:64) u11 P4 (33:32) u13 M6 (26:27) u17 aug8 (2176:2187) u19 m3 (513:512) u23 aug4 (736:729)

u29 G#-39 (261:256) u31 P8 (31:32) u37 M2 (37:36) u41 M3 (82:81) u43 P4 (129:128) aug4 (752:729)

The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes
with dedicated microtonal accidental symbols for primes 5 through 47

revised by Marc Sabat & Thomas Nicholson

in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above ♯A

8¹
partial interval alteration (81:80)
5° M3 (64:63)
7° m7 (32:33)
Fifth 2:3
11° P4 (32:33)
13° M6 (27:26)
17° aug8 (2187:2176)
19° m3 (512:513)
23° aug4 (729:736)
-27 F+41 -31 -12 A♭+45° A♯-47 -45 -2 C+42 -14 -29 -10 47° aug4 (729:752)
29° m7 (256:261)
31° P8 (32:31)
37° M2 (36:37)
41° M3 (81:82)
43° P4 (128:129)
E-34 E+38 (729:752)

Standard utonal notation below ♯E

+2 8¹ +2
u5 M3 (80:81)
u7 m7 (63:64)
+16 +33 +2 +16
-2 -49 u11 P4 (33:32)
-49 u13 M6 (26:27)
+16 +33 +14 +2
G♯-39 +4 C♯-41 +16 +31
+29 G♯-39 +33 +14 F-43 +2 +47 +16 +31
-4 -28 -51 -3 -2 -49 -27 -10 -49 -26 A+36 u47 A-36
u29 m7 (261:256)
u31 P8 (31:32)
u37 M2 (37:36)
u41 M3 (82:81)
u43 P4 (129:128)
u47 aug4 (752:729)

The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1-49 notated by modifications of Pythagorean notes *with dedicated microtonal accidental symbols for primes 5 through 47*

revised by Marc Sabat & Thomas Nicholson

in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol, cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above $\natural A$

Standard octonal notation above A

Standard utonal notation below $\natural E$

Sheet music for a two-part composition. The top staff is in treble clef and the bottom staff is in bass clef. The music is in 8/8 time, indicated by the first measure. Various intervals and ratios are labeled below the notes, such as $\text{M}3$, $\text{P}4$, $\text{aug}8$, and $\text{aug}4$. The bottom staff continues the sequence with notes and labels like $\text{m}7$, $\text{P}8$, $\text{M}2$, $\text{M}3$, $\text{P}4$, and $\text{aug}4$.

The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

and the row also contains chords, i.e. the Major Triad 4:5:6

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol, cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above $\natural A$

8^o partial interval alteration (81:80) 5^o M3 (64:63) 7^o m7 (32:33) 11^o P4 (32:33) 13^o M6 (27:26) F+41 -12 17^o aug8 (2187:2176) 19^o m3 (512:513) 23^o aug4 (729:736)

-27 F+41 -31 -12 Ab+45^o 29^o m7 (256:261) 31^o P8 (32:31) -45 37^o M2 (36:37) -2 C+42 41^o M3 (81:82) 43^o P4 (128:129) -14 47^o aug4 (729:752) -10 +28 E-34 +2 E+38

Standard utonal notation below $\natural E$

+2 8^o +2 u5 M3 (80:81) u7 m7 (63:64) +16 +33 +2 -2 -49 G#-39 +16 +33 +14 +2 -3 -2 +4 +16 +31 -49 -26

u11 P4 (33:32) u13 M6 (26:27) u17 aug8 (2176:2187) u19 m3 (513:512) u23 aug4 (736:729) +29 G#-39 +33 +14 F-43 +2 +47 +4 C#-41 +16 +31 +12 +27 -10 -49 -26 A+36 u47 A-36

u29 m7 (261:256) u31 P8 (31:32) u37 M2 (37:36) u41 M3 (82:81) u43 P4 (129:128) aug4 (752:729)

revised by Marc Sabat & Thomas Nicholson

in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz

The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

Major Triad inversion 5:6:8

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol, cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above $\natural A$

Standard octonal notation above A

Standard utonal notation below $\natural E$

Sheet music for a composition featuring two staves. The top staff is in treble clef and the bottom staff is in bass clef. The music is in 8/8 time. Various intervals and ratios are labeled below the notes. The top staff includes labels: u5 , M3 , $(80:81)$, u7 , m7 , $(63:64)$, P4 , $(33:32)$, u11 , M6 , $(26:27)$, u13 , aug8 , $(2176:2187)$, u17 , m3 , $(513:512)$, u19 , aug4 , $(736:729)$, u23 , and aug4 . The bottom staff includes labels: $\text{G}^{\#}\text{-39}$, F-43 , u29 , m7 , $(261:256)$, +29 , +33 , +14 , +2 , +47 , +4 , $\text{C}^{\#}\text{-41}$, +16 , +31 , +12 , A+36 , A-36 , u31 , P8 , $(31:32)$, u37 , M2 , $(37:36)$, u41 , M3 , $(82:81)$, u43 , P4 , $(129:128)$, u47 , and aug4 , $(752:729)$.

cc 2020 Plainsound Music Edition

The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

Minor Triad 10:12:15

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol, cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above ♯A

8^o partial interval alteration (81:80)

5^o M3 (64:63)

7^o m7 (32:33)

11^o P4 (32:33)

13^o M6 (27:26)

17^o aug8 (2187:2176)

19^o m3 (512:513)

23^o aug4 (729:736)

29^o m7 (256:261)

31^o P8 (32:31)

37^o M2 (36:37)

41^o M3 (81:82)

43^o P4 (128:129)

47^o aug4 (729:752)

Standard utonal notation below ♯E

u5 M3 (80:81)

u7 m7 (63:64)

u11 P4 (26:27)

u13 M6 (26:27)

u17 aug8 (2176:2187)

u19 m3 (513:512)

u23 aug4 (736:729)

u29 m7 (261:256)

u31 P8 (31:32)

u37 M2 (37:36)

u41 M3 (82:81)

u43 P4 (129:128)

u47 aug4 (752:729)

revised by Marc Sabat & Thomas Nicholson

in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz

The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

Diminished Triad 5:6:7

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol, cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above ♯A

8^o partial interval alteration (81:80)

5° M3 (64:63)

7° m7 (32:33)

11° P4 (32:33)

13° M6 (27:26)

17° aug8 (2187:2176)

19° m3 (512:513)

23° aug4 (729:736)

29° m7 (256:261)

31° P8 (32:31)

37° M2 (36:37)

41° M3 (81:82)

43° P4 (128:129)

47° aug4 (729:752)

Standard utonal notation below ♯E

u5 M3 (80:81)

u7 m7 (63:64)

u11 P4 (33:32)

u13 M6 (26:27)

u17 aug8 (2176:2187)

u19 m3 (513:512)

u23 aug4 (736:729)

u29 m7 (261:256)

u31 P8 (31:32)

u37 M2 (37:36)

u41 M3 (82:81)

u43 P4 (129:128)

u47 aug4 (752:729)

revised by Marc Sabat & Thomas Nicholson

in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz

The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes
with dedicated microtonal accidental symbols for primes 5 through 47

and aggregates of four or more notes, i.e. the Dominant 7th

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above $\natural A$

8 partial interval alteration (81:80) 5° m7 (64:63) 11° P4 (32:33) 13° M6 (27:26) 17° aug8 (2187:2176) 19° m3 (512:513) 23° aug4 (729:736)

-27 F+41 -31 -12 A \flat +45° 29° m7 (256:261) 31° P8 (32:31) -45 37° M2 (36:37) -2 C+42 41° M3 (81:82) 43° P4 (128:129) 47° aug4 (729:752) -14 -29 -10

Standard utonal notation below $\natural E$

+2 8 -2 2 +16 +33 +2 +16 +2 +33 +14 +2 +4 +16 +31 -4 G \sharp -39 -2 -49 u5 M3 (80:81) u7 m7 (63:64) u11 P4 (33:32) u13 M6 (26:27) u17 aug8 (2176:2187) u19 m3 (513:512) u23 aug4 (736:729) -51 +47 +4 C \sharp -41 +16 +31 +12 -27 -10 -49 -26 A+36 u47 A-36

+29 G \sharp -39 +33 +14 F-43 +2 +47 +4 C \sharp -41 +16 +31 +12 -27 -10 -49 -26 A+36 u47 A-36

-4 -28 -51 -3 -2 -49 u29 m7 (261:256) u31 P8 (31:32) u37 M2 (37:36) u41 M3 (82:81) u43 P4 (129:128) aug4 (752:729)

revised by Marc Sabat & Thomas Nicholson

in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz

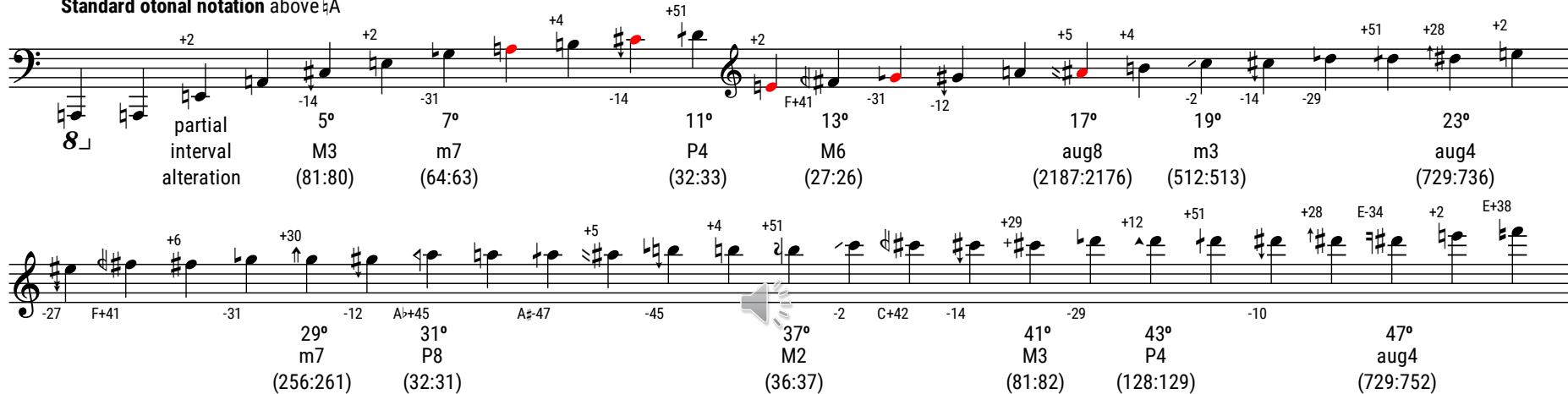
The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes
with dedicated microtonal accidental symbols for primes 5 through 47

Dominant 7 flat 9 using 17° !

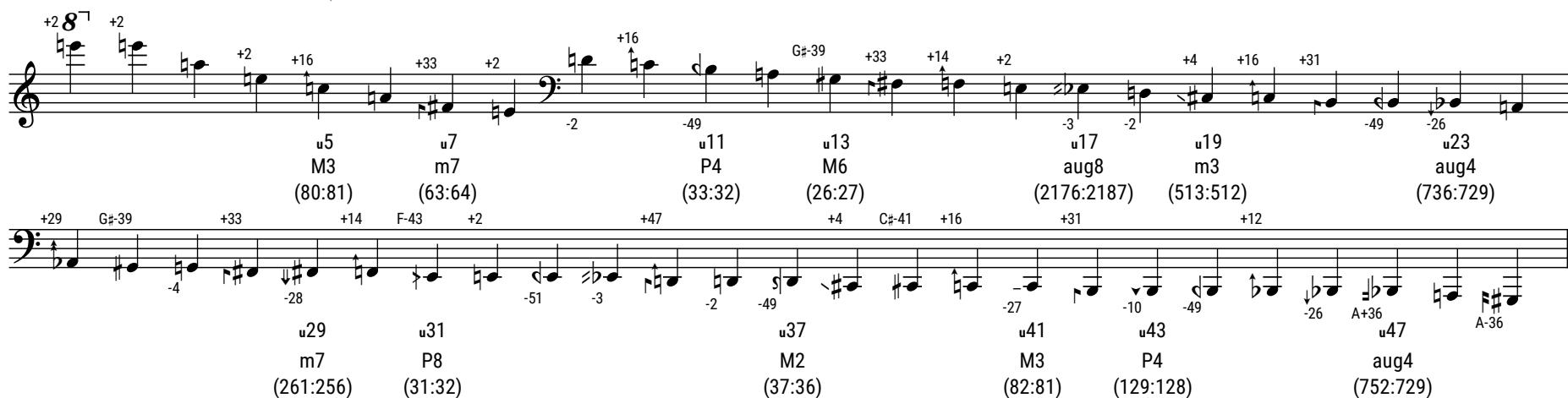
Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above ♯A



A musical score for standard otonal notation above ♯A. The score consists of two staves. The top staff is in bass clef and the bottom staff is in treble clef. The notation uses Pythagorean note names (A, B, C, D, E, F, G) with various accidentals (sharps, flats, naturals) and microtonal symbols. Numerical values above the notes indicate the amount of modification (e.g., +2, +4, +51 cents) and below the notes indicate deviations from A = 0 cents. The score includes labels for partial interval alteration, intervals (e.g., 8, 5°, 7°, 11°, 13°, 17°, 19°, 23°), and ratios (e.g., (81:80), (64:63), (32:33), (27:26), (2187:2176), (512:513), (729:736)). A central note is labeled 37° M2 (36:37).

Standard utonal notation below ♯E



A musical score for standard utonal notation below ♯E. The score consists of two staves. The top staff is in treble clef and the bottom staff is in bass clef. The notation uses Pythagorean note names with accidentals and microtonal symbols. Numerical values above the notes indicate the amount of modification and below the notes indicate deviations from A = 0 cents. The score includes labels for intervals (e.g., u5, u7, u11, u13, u17, u19, u23), ratios (e.g., (80:81), (63:64), (33:32), (26:27), (2176:2187), (513:512), (736:729)), and note names (e.g., G♯-39, F-43, C♯-41, A+36, A-36).

revised by Marc Sabat & Thomas Nicholson

in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz

The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

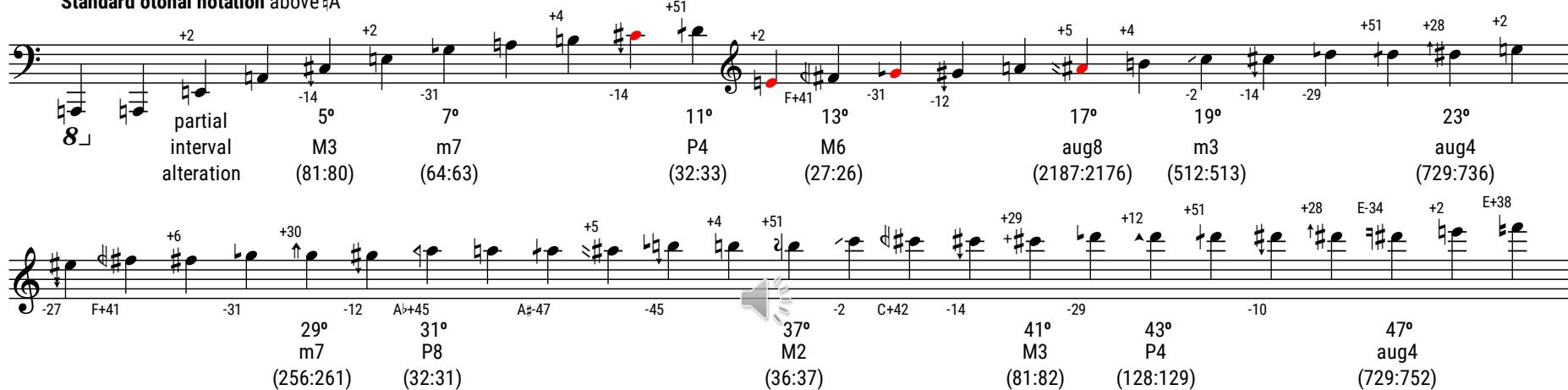
Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

Diminished Chord

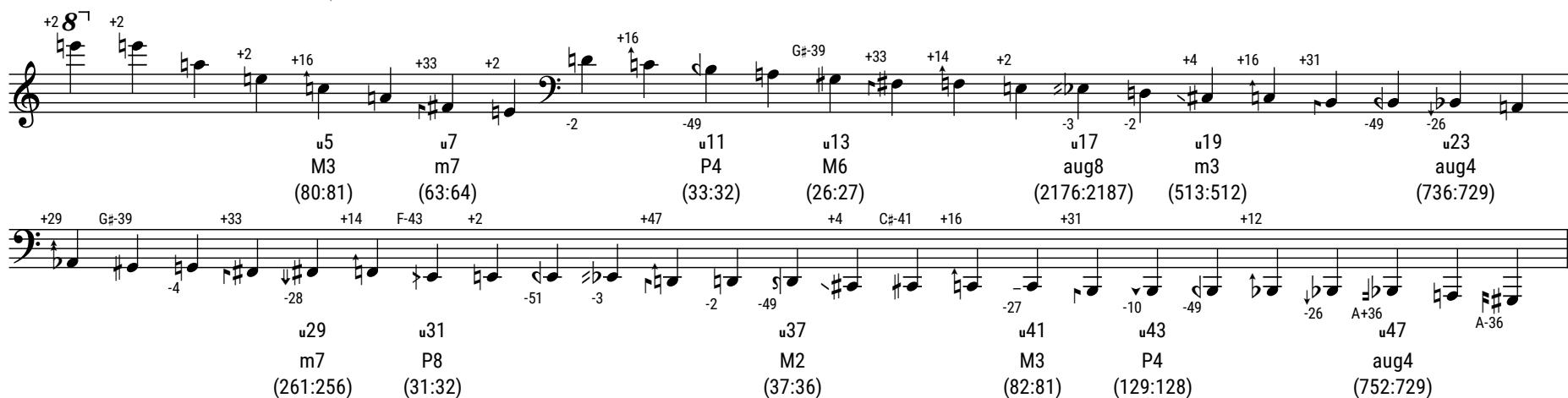
Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above ♯A



A musical score for standard otonal notation above ♯A. The score consists of two staves. The top staff is in bass clef and the bottom staff is in treble clef. The notation uses Pythagorean note heads with various accidentals (sharps, flats, naturals) and note heads with a diagonal line through them. Numerical values above the notes indicate cent deviations from Pythagorean tuning. Below the notes, labels provide the name of the interval, its ratio, and its equivalent cents value. The score includes intervals such as partial interval alteration (81:80), M3 (5°), m7 (7°), P4 (11°), M6 (13°), aug8 (17°), m3 (19°), aug4 (23°), and E+38 (279:736). The bottom staff continues with intervals like 29° m7 (256:261), 31° P8 (32:31), 37° M2 (36:37), 41° M3 (81:82), 43° P4 (128:129), and 47° aug4 (729:752).

Standard utonal notation below ♯E



A musical score for standard utonal notation below ♯E. The score consists of two staves. The top staff is in treble clef and the bottom staff is in bass clef. The notation uses Pythagorean note heads with various accidentals and note heads with a diagonal line through them. Numerical values above the notes indicate cent deviations from Pythagorean tuning. Below the notes, labels provide the name of the interval, its ratio, and its equivalent cents value. The score includes intervals such as u5 M3 (80:81), u7 m7 (63:64), u11 P4 (33:32), u13 M6 (26:27), aug8 (2176:2187), m3 (513:512), and aug4 (736:729). The bottom staff continues with intervals like u29 m7 (261:256), u31 P8 (31:32), u37 M2 (37:36), u41 M3 (82:81), u43 P4 (129:128), and aug4 (752:729).

revised by Marc Sabat & Thomas Nicholson

in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz

The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

and new microtonal chords, i.e. Septimal Chord 6:7:8:9

revised by Marc Sabat & Thomas Nicholson

in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above $\natural A$

8¹ partial interval alteration (81:80) 5° m7 (64:63) 11° P4 (32:33) 13° M6 (27:26) 17° aug8 (2187:2176) 19° m3 (512:513) 23° aug4 (729:736)

-27 F+41 -31 -12 Ab+45° A#-47 -45 -2 C+42 -14 -29 -10 47° aug4 (729:752)

29° m7 (256:261) 31° P8 (32:31) 37° M2 (36:37) 41° M3 (81:82) 43° P4 (128:129) 47° aug4 (729:752)

Standard utonal notation below $\natural E$

+2 8¹ +2

u5 M3 (80:81) u7 m7 (63:64) -2 -49 u11 P4 (33:32) u13 M6 (26:27) +16 G#-39 +33 +14 +2 +4 +16 +31

+29 G#-39 +33 +14 F-43 +2 +47 +4 C#-41 +16 +31 +12 +4 +16 +31

-4 -28 -51 -3 -2 -49 -27 -10 -49 -26 A+36 u47 A-36

u29 m7 (261:256) u31 P8 (31:32) u37 M2 (37:36) u41 M3 (82:81) u43 P4 (129:128) aug4 (752:729)

The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes
with dedicated microtonal accidental symbols for primes 5 through 47

compare the Septimal Chord 6:7:8:9 an octave lower

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

revised by Marc Sabat & Thomas Nicholson

in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz

Standard otonal notation above $\natural A$

A musical score for the Septimal Chord 6:7:8:9 an octave lower in standard otonal notation above natural A. The score consists of two staves. The top staff is in bass clef and the bottom staff is in treble clef. The notation uses Pythagorean note names (A, B, C, D, E, F, G) with various accidentals (sharps, flats, naturals) and note heads (black, white, red) to represent microtonal modifications. Numerical values above the notes indicate cents deviations from Pythagorean tuning. The score includes labels for intervals and ratios, such as 'partial interval alteration' (81:80), 'M3' (5°), 'm7' (7°), 'P4' (11°), 'M6' (13°), 'aug8' (17°), 'm3' (19°), 'aug4' (23°), 'M2' (37°), 'P4' (41°), 'M3' (43°), and 'aug4' (47°). Ratios like (81:80), (64:63), (32:33), (27:26), (2187:2176), (512:513), (729:736), (256:261), (32:31), (36:37), (81:82), (128:129), and (729:752) are also provided.

Standard utonal notation below $\natural E$

A musical score for the Septimal Chord 6:7:8:9 an octave lower in standard utonal notation below natural E. The score consists of two staves. The top staff is in treble clef and the bottom staff is in bass clef. The notation uses Pythagorean note names with accidentals and note heads. Numerical values above the notes indicate cents deviations. The score includes labels for intervals and ratios, such as 'u5' (M3), 'u7' (m7), 'P4' (31°), 'M6' (37°), 'aug8' (41°), 'm3' (43°), 'aug4' (47°), 'M2' (37°), 'P4' (41°), 'M3' (43°), and 'aug4' (47°). Ratios like (80:81), (63:64), (33:32), (26:27), (2176:2187), (513:512), (736:729), (261:256), (31:32), (37:36), (82:81), (129:128), and (752:729) are also provided.

The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

Undecimal Chord 8:9:11:12

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

Standard otonal notation above ♯A

8 partial interval alteration (81:80)

5° M3 (64:63)

7° m7 (32:33)

11° P4 (32:33)

13° M6 (27:26)

17° aug8 (2187:2176)

19° m3 (512:513)

23° aug4 (729:736)

29° m7 (256:261)

31° P8 (32:31)

37° M2 (36:37)

41° M3 (81:82)

43° P4 (128:129)

47° aug4 (729:752)

Standard utonal notation below ♯E

u5 M3 (80:81)

u7 m7 (63:64)

u11 P4 (33:32)

u13 M6 (26:27)

u17 aug8 (2176:2187)

u19 m3 (513:512)

u23 aug4 (736:729)

u29 m7 (261:256)

u31 P8 (31:32)

u37 M2 (37:36)

u41 M3 (82:81)

u43 P4 (129:128)

u47 aug4 (752:729)

revised by Marc Sabat & Thomas Nicholson

in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz



The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020

Harmonic / Subharmonic series 1–49 notated by modifications of Pythagorean notes

with dedicated microtonal accidental symbols for primes 5 through 47

Tridecimal Chord 10:12:13:15

Ratios represent the amount of modification of the Pythagorean notes by each additional symbol,
cents indications are deviations that would be shown on a tuning meter with A = 0 cents

revised by Marc Sabat & Thomas Nicholson

in collaboration with Wolfgang von Schweinitz, Catherine Lamb and M.O. Abbott
building upon the original HEJI devised by Marc Sabat and Wolfgang von Schweinitz

Standard otonal notation above ♯A

8 partial interval alteration (81:80) M3 (5°) m7 (7°) P4 (11°) M6 (13°) aug8 (17°) m3 (19°) aug4 (23°) E+38 (27°)

29° m7 (256:261) 31° P8 (32:31) 37° M2 (36:37) 41° M3 (81:82) 43° P4 (128:129) 47° aug4 (729:752)

Standard utonal notation below ♯E

u5 M3 (80:81) u7 m7 (63:64) P4 (33:32) M6 (26:27) aug8 (2176:2187) m3 (513:512) aug4 (736:729)

u29 m7 (261:256) u31 P8 (31:32) M2 (37:36) M3 (82:81) P4 (129:128) aug4 (752:729)



Harmony by numbers: tuning in practice

- How can a performer learn to hear and play such tunings accurately?



Harmony by numbers: tuning in practice

- How can a performer learn to hear and play such tunings accurately?
by perceiving acoustic and psychoacoustic phenomena



Harmony by numbers: difference (Tartini) tones

When two frequencies interact in the same medium (air, eardrum, brain, etc.) they produce *combination tones*, i.e., distortion at various linear combinations of their frequencies:

$$m \cdot F_1 + n \cdot F_2$$

where m and n are integers. The combination tone most easily perceived is the *difference tone*.

Harmony by numbers: difference (Tartini) tones

When two frequencies interact in the same medium (air, eardrum, brain, etc.) they produce *combination tones*, i.e., distortion at various linear combinations of their frequencies:

$$m \cdot F_1 + n \cdot F_2$$

where m and n are integers. The combination tone most easily perceived is the *difference tone*.

In many simple ratios between partials, like 1:2 (octave), 2:3 (fifth), 3:4 (fourth), 4:5 (major third), and 3:5 (major sixth), the difference tone reinforces their common fundamental, making a clear, consonant harmony.



Harmony by numbers: difference (Tartini) tones

Violin

8/5 13/8 5/3

3° 5° 2°

Difference Tones & Fundamentals



Harmony by numbers: common partials

When two frequencies are tuned rationally as $a : b$, expressed in lowest terms, they will share some common partials. Namely, the b th partial of a is equal to the a th partial of b . The lowest common partial of the two pitches may be calculated as the *product* of a and b .

$$\text{Common partial} = a \cdot b$$



Harmony by numbers: tuning by listening to beating

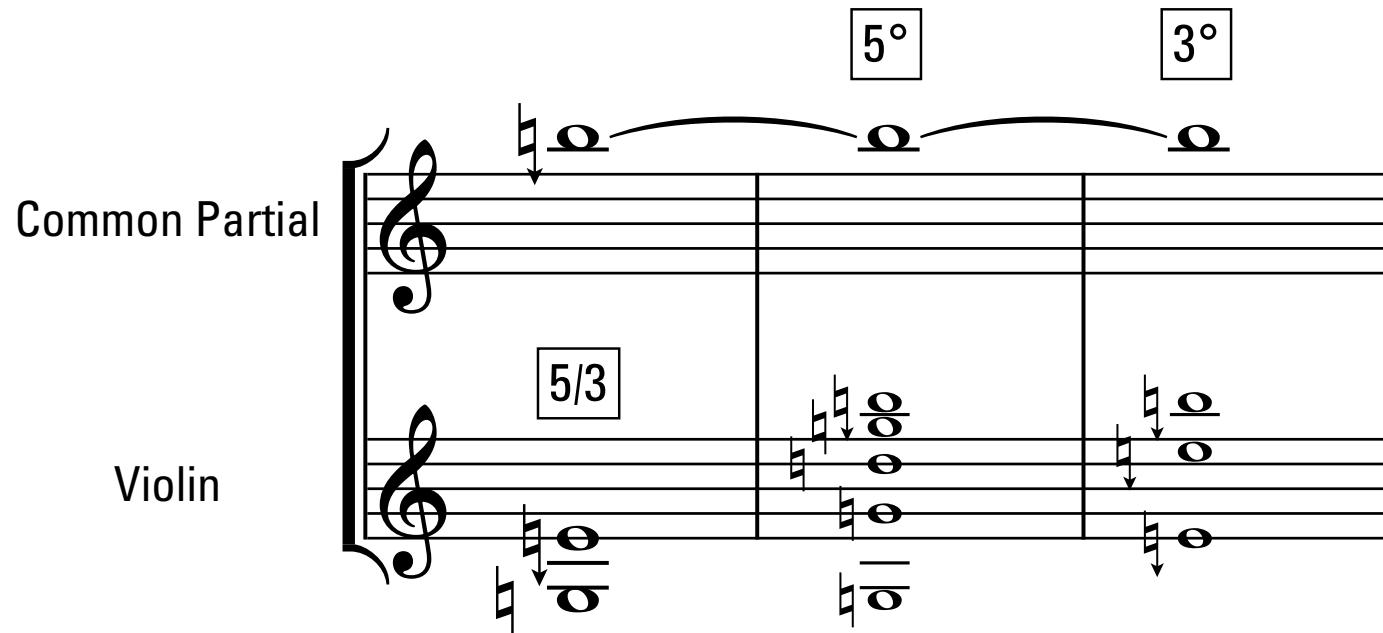
When two frequencies are tuned rationally as $a : b$, expressed in lowest terms, they will share some common partials. Namely, the b th partial of a is equal to the a th partial of b . The lowest common partial of the two pitches may be calculated as the *product* of a and b .

$$\text{Common partial} = a \cdot b$$

Beating at the common partial is one of the most commonly perceived phenomena when musicians learn to tune intervals. Eliminating beating to produce so-called “pure” intervals is one way to make JI sounds.



Harmony by numbers: tuning by listening to beating





Harmony by numbers: degrees of tuneability

Small number ratios produce intervals that may be easily identified and tuned. As numbers comprising a ratio become larger, pitch distances to the interval's common fundamental and common partial also increase. Eventually, such intervals are no longer precisely recognisable, and therefore, no longer tuneable by ear.

Harmony by numbers: chords, melodies

Small number ratios produce intervals that may be easily identified and tuned. As numbers comprising a ratio become larger, pitch distances to the interval's common fundamental and common partial also increase. Eventually, such intervals are no longer precisely recognisable, and therefore, no longer tuneable by ear.

In my music for live musicians, I find it useful to work with tuneability, to focus on the experience of hearing and playing the unique sonorities and resonances of rational intonation. Concentrating on intervals and chords making melodies by means of counterpoint, rather than using fixed scales or a single row, allows a free flow of sounds with shifting fundamentals and changing points of reference.



Harmony by numbers: applying JI to music repertoire

- tuning of the open strings



Harmony by numbers: applying JI to music repertoire

- tuning of the open strings *in perfect fifths*



Harmony by numbers: applying JI to music repertoire

- tuning of the open strings *in perfect fifths*
- tuning of major, minor, and diminished chords



Harmony by numbers: applying JI to music repertoire

- tuning of the open strings *in perfect fifths*
- tuning of major, minor, and diminished chords
1:3:5 (major)



Harmony by numbers: applying JI to music repertoire

- tuning of the open strings *in perfect fifths*
- tuning of major, minor, and diminished chords
1:3:5 (major); 3:5:15 (minor)



Harmony by numbers: applying JI to music repertoire

- tuning of the open strings *in perfect fifths*
- tuning of major, minor, and diminished chords
1:3:5 (major); 3:5:15 (minor); 3:5:7 (diminished)



Harmony by numbers: applying JI to music repertoire

- tuning of the open strings *in perfect fifths*
- tuning of major, minor, and diminished chords
1:3:5 (major); 3:5:15 (minor); 3:5:7 (diminished)
- tuning of scales and modes



Harmony by numbers: applying JI to music repertoire

- tuning of the open strings *in perfect fifths*
- tuning of major, minor, and diminished chords
1:3:5 (major); 3:5:15 (minor); 3:5:7 (diminished)
- tuning of scales and modes, *major*



Harmony by numbers: applying JI to music repertoire

- tuning of the open strings *in perfect fifths*
- tuning of major, minor, and diminished chords
1:3:5 (major); 3:5:15 (minor); 3:5:7 (diminished)
- tuning of scales and modes, *major and minor*



Harmony by numbers: applying JI to music repertoire

- tuning of the open strings *in perfect fifths*
- tuning of major, minor, and diminished chords
1:3:5 (major); 3:5:15 (minor); 3:5:7 (diminished)
- tuning of scales and modes, *major and minor*
- modulation?



Harmony by numbers: applying JI to music repertoire

- tuning of the open strings *in perfect fifths*
- tuning of major, minor, and diminished chords
1:3:5 (major); 3:5:15 (minor); 3:5:7 (diminished)
- tuning of scales and modes, *major and minor*
- modulation?



Harmony by numbers: applying JI to music

- tuning of the open strings *in perfect fifths*
- tuning of major, minor, and diminished chords
1:3:5 (major); 3:5:15 (minor); 3:5:7 (diminished)
- tuning of scales and modes, *major and minor*
- modulation?



Harmony by numbers: applying JI to music

- tuning of the open strings *in perfect fifths*
- tuning of major, minor, and diminished chords
1:3:5 (major); 3:5:15 (minor); 3:5:7 (diminished)
- tuning of scales and modes, *major and minor*
- modulation!



Harmony by numbers: applying JI to music

- tuning of the open strings *in perfect fifths*
- tuning of major, minor, and diminished chords
1:3:5 (major); 3:5:15 (minor); 3:5:7 (diminished)
- tuning of scales and modes, *major and minor*
- why limit the primes to 7° (and occasionally a 17° as flat 9th)?



Harmony by numbers: applying JI to music repertoire

- tuning of the open strings *in perfect fifths*
- tuning of major, minor, and diminished chords
1:3:5 (major); 3:5:15 (minor); 3:5:7 (diminished)
- tuning of scales and modes, *major and minor*
- why limit the primes to 7° (and occasionally a 17° as flat 9th)?
- why two violins? why not electronics, or more compositional interventions?



Harmony by numbers: applying JI to music repertoire

- tuning of the open strings *in perfect fifths*
- tuning of major, minor, and diminished chords
1:3:5 (major); 3:5:15 (minor); 3:5:7 (diminished)
- tuning of scales and modes, *major and minor*
- why limit the primes to 7° (and occasionally a 17° as flat 9th)?
- why two violins? why not electronics, or more compositional interventions?
- when and how to make choices between pure vertical consonances (introducing more microtonal variation) vs. maintaining melodic simplicity?



Streams barely in winter

three beginnings for Walter

Marc Sabat

1. Cold

Violin 1

Violin 2

Cello/Bass

3°/IV

2°/III

3°/IV 4°/IV

3°/III

II

,

attacca

*Sonata Prima per Violino Solo senza Basso
Sei Bach-Intonazioni, Ia*



Sonata Prima a Violino Solo senza Basso

Sei Bach-Intonazioni per Violino Solo, la

Johann Sebastian Bach

intonation and 2nd voice composed by Marc Sabat

Violin (solo)

Adagio

4°/III 2°/IV

Violin 2 (bordun)

sempre sotto voce

V I

tr

2b

3

E-natural
(not Eb) in MS

tr

4°/IV

V

2°/IV

2°/III

Sonata Prima a Violino Solo senza Basso

Bach begins by showing the notes, in this case a diatonic scale. The pitches are tuned as simple ratios from G and D.

Sei Bach-Intonazioni per Violino Solo, la

Johann Sebastian Bach

intonation and 2nd voice composed by Marc Sabat

Violin (solo)

Adagio

4°/III 2°/IV

sempre sotto voce

V I

2b

E-natural
(not Eb) in MS

4°/IV V 2°/IV 2°/III

tr

Sonata Prima a Violino Solo senza Basso

Sei Bach-Intonazioni per Violino Solo, la

Johann Sebastian Bach

intonation and 2nd voice composed by Marc Sabat

C is first tuned as a 3/2 below G, then as a 7°/D

Violin (solo)

Adagio

4°/III 2°/IV

Violin 2 (bordun)

sempre sotto voce

V I

tr

2b

3

E-natural
(not Eb) in MS

tr

4°/IV

V

2°/IV

2°/III



Streams barely in winter (2019) 1. *Cold* 2. *Sun* 3. *Stones*
three beginnings for Walter played before Bach for two violins

Sei Bach-Intonazioni per Violino Solo (2000–17) Ia IIa IIIa
version in just intonation for violin solo with violin bordun counterpoint

Chords, melodies: a look at harmony by numbers
Sara Cubarsi and Xenia Gogu, violins

Streams barely in winter

three beginnings for Walter

Marc Sabat

1. Cold

Violin 1

Violin 2

Cold

attacca
Sonata Prima per Violino Solo senza Basso
Sei Bach-Intonazioni, Ia

Sonata Prima a Violino Solo senza Basso

Sei Bach-Intonazioni per Violino Solo, Ia

Johann Sebastian Bach

intonation and 2nd voice composed by Marc Sabat

Violin (solo)

Adagio

4°/III 2°/IV

sempre sotto voce

E-natural (not Eb) in MS

tr

2b

4°/IV

3

2°/IV

2°/III

tr

4

tr

2°/II

2°/IV

2

6

v

v

8

tr

v

9b

10

2°/IV

v

11

3

13

V

14b

15

4

16

2°/IV

18

2°/III

20

3°/IV

4°/IV

5°/IV

Fuga

Allegro

E-natural
(not Eb) in MS

5

4°/III

2°/III

4°/IV

2°/IV

2°/III

8

2°/IV

6

11

3°/IV 4°/IV 2°/III

Measures 11, 12, and 13 of a musical score. The music is in 12/8 time. The key signature changes from 3°/IV (one sharp) to 4°/IV (two sharps) to 2°/III (one sharp). The score consists of two staves. The top staff has a treble clef and the bottom staff has a bass clef. The music features eighth-note and sixteenth-note patterns with various rests and dynamic markings like accents and staccato dots.

14

2°/IV 3°/III 4°/IV 4°/III 4°/II 4°/IV

Measures 14, 15, and 16 of the musical score. The key signature changes back to 2°/IV (no sharps or flats). The score continues with eighth-note and sixteenth-note patterns, including a prominent bass line in the bottom staff.

18

2°/III 2°/II

Measures 18, 19, and 20 of the musical score. The key signature changes to 2°/III (no sharps or flats). The score maintains its eighth-note and sixteenth-note patterns, with a focus on the bass line.

22

V 3°/II V

Measures 22, 23, and 24 of the musical score. The key signature changes to 3°/II (two sharps). The score concludes with a final cadence, indicated by a 'V' symbol.

26

3°/III

5°/IV

2°/II

4°/IV

2°/III

7

30

4°/II

34

5°/II

8°/III

4°/I

39

2°/III

2°/II

43

2°/IV

2°/III

46

2°/IV

2°/IV

49

2°/IV

2°/III

52

2°/III

V

56

60

63

3°/III

66

4°/IV

5°

10

69

2°/III

2°/III

2°/III

2°/III

2°/III

2°/III

2°/IV

3°/II

4°/III

4°/II

4°/IV

3°/IV

3°/III

2°/IV

83

87

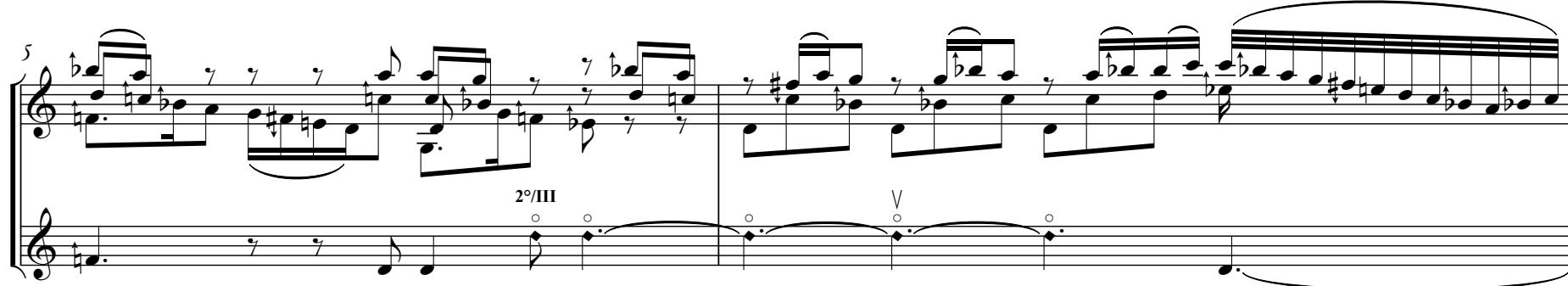
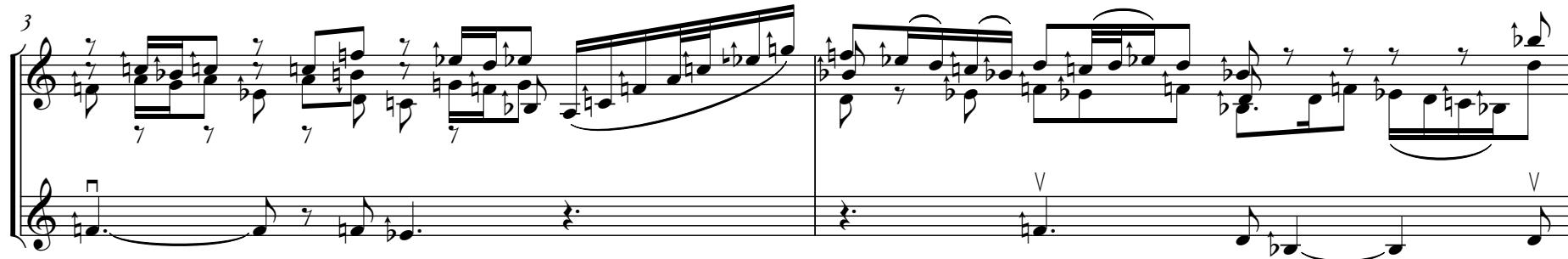
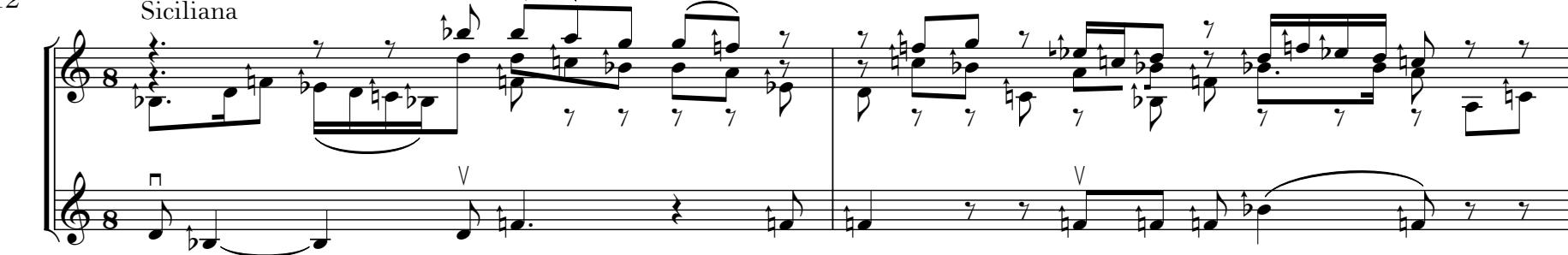
2°/IV

90

93

12

Siciliana



13

7

9

3°/IV

4°/IV

11

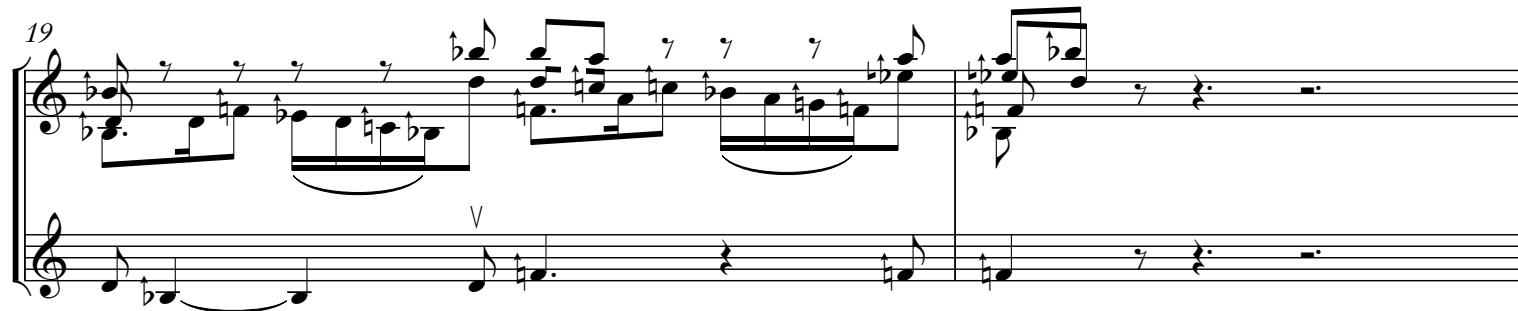
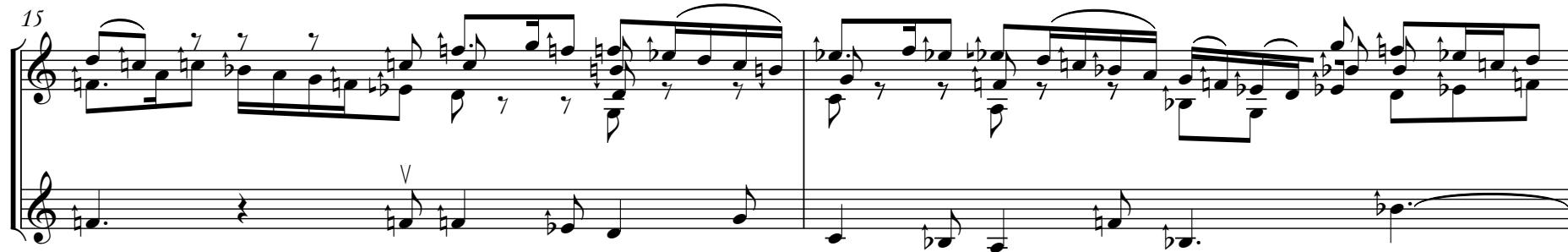
13

V

V

13

14



Musical score for two staves, Presto tempo, 3/8 time, 4 sharps. The score consists of four systems of music. The top staff uses a treble clef and the bottom staff uses a bass clef. The music is in common time for the first three systems and 3/8 time for the fourth system. The score features various note heads, stems, and beams, with some notes having upward or downward arrows indicating specific performance techniques. Measure numbers 15, 8, 15, and 22 are marked above the staves.

Presto

8

15

22

16

29

36

44

52

Presto

8

15

22

16

29

36

44

52

A musical score for two staves, featuring measures 59, 66, 73, and 80. The top staff uses a treble clef and the bottom staff uses a bass clef. Measure 59 (measures 1-4) starts with a sixteenth-note pattern in the treble staff, followed by eighth-note pairs in the bass staff. Measure 66 (measures 1-4) shows eighth-note pairs in the treble staff and sixteenth-note patterns in the bass staff. Measure 73 (measures 1-4) features eighth-note pairs in the treble staff and sixteenth-note patterns in the bass staff. Measure 80 (measures 1-4) concludes the page with eighth-note pairs in the treble staff and sixteenth-note patterns in the bass staff. The score includes various accidentals (sharps, flats, naturals) and slurs.

18

87

Measures 87-92: Treble staff: eighth-note patterns with accidentals. Bass staff: eighth-note patterns with accidentals. Measures 93-98: Treble staff: eighth-note patterns with accidentals. Bass staff: eighth-note patterns with accidentals.

94

Measures 94-99: Treble staff: eighth-note patterns with accidentals. Bass staff: eighth-note patterns with accidentals. Measures 100-105: Treble staff: eighth-note patterns with accidentals. Bass staff: eighth-note patterns with accidentals.

101

Measures 101-106: Treble staff: eighth-note patterns with accidentals. Bass staff: eighth-note patterns with accidentals. Measures 107-112: Treble staff: eighth-note patterns with accidentals. Bass staff: eighth-note patterns with accidentals.

108

Measures 108-113: Treble staff: eighth-note patterns with accidentals. Bass staff: eighth-note patterns with accidentals. Measures 114-119: Treble staff: eighth-note patterns with accidentals. Bass staff: eighth-note patterns with accidentals. Measure 120: Treble staff: eighth-note patterns with accidentals. Bass staff: eighth-note patterns with accidentals.

3°/IV 4°/IV 3°/III

Musical score for piano, page 19, measures 115-121. The score consists of two staves. The top staff uses a treble clef and the bottom staff uses a bass clef. Measure 115 starts with a sixteenth-note pattern in the treble staff, followed by eighth-note pairs in the bass staff. Measures 116-121 continue this pattern with various note heads and stems, including a prominent eighth-note in the bass staff of measure 121.

Musical score for piano, page 19, measures 122-128. The score consists of two staves. The top staff uses a treble clef and the bottom staff uses a bass clef. Measures 122-128 show a continuation of the sixteenth-note and eighth-note patterns from the previous measures, with the bass staff providing harmonic support through sustained notes and rhythmic patterns.

Musical score for piano, page 19, measures 129-135. The score consists of two staves. The top staff uses a treble clef and the bottom staff uses a bass clef. Measures 129-135 feature a mix of sixteenth-note patterns and sustained notes. The bass staff includes a circled '2°/III' symbol, indicating a harmonic progression. Measures 134-135 conclude with a final harmonic cadence.

16

29

36

44

52

A musical score for two staves, featuring measures 59, 66, 73, and 80. The top staff uses a treble clef and the bottom staff uses a bass clef. Measure 59 (measures 1-4) starts with a sixteenth-note pattern in the treble staff, followed by eighth-note pairs in the bass staff. Measure 66 (measures 1-4) shows eighth-note pairs in the treble staff and sixteenth-note patterns in the bass staff. Measure 73 (measures 1-4) features eighth-note pairs in the treble staff and sixteenth-note patterns in the bass staff. Measure 80 (measures 1-4) concludes the page with eighth-note pairs in the treble staff and sixteenth-note patterns in the bass staff. The score includes various accidentals (sharps, flats, naturals) and slurs.

18

87

Measures 87-92: Treble staff: eighth-note patterns with accidentals. Bass staff: eighth-note patterns with accidentals. Measures 93-98: Treble staff: eighth-note patterns with accidentals. Bass staff: eighth-note patterns with accidentals.

94

Measures 94-99: Treble staff: eighth-note patterns with accidentals. Bass staff: eighth-note patterns with accidentals. Measures 100-105: Treble staff: eighth-note patterns with accidentals. Bass staff: eighth-note patterns with accidentals.

101

Measures 101-106: Treble staff: eighth-note patterns with accidentals. Bass staff: eighth-note patterns with accidentals. Measures 107-112: Treble staff: eighth-note patterns with accidentals. Bass staff: eighth-note patterns with accidentals.

108

Measures 108-113: Treble staff: eighth-note patterns with accidentals. Bass staff: eighth-note patterns with accidentals. Measures 114-119: Treble staff: eighth-note patterns with accidentals. Bass staff: eighth-note patterns with accidentals. Measure 120: Treble staff: eighth-note patterns with accidentals. Bass staff: eighth-note patterns with accidentals.

3°/IV 4°/IV 3°/III

115

Measures 115-121 of a piano score. The top staff uses a treble clef and the bottom staff uses a bass clef. The music consists of six measures of sixteenth-note patterns. Measure 115 starts with a sixteenth-note pattern on the top staff, followed by a bass note on the bottom staff. Measures 116-121 follow a similar pattern of sixteenth-note groups and bass notes, with some variations in the top staff's notes and rests.

122

Measures 122-128 of a piano score. The top staff uses a treble clef and the bottom staff uses a bass clef. The music consists of seven measures of sixteenth-note patterns. Measure 122 starts with a sixteenth-note pattern on the top staff, followed by a bass note on the bottom staff. Measures 123-128 follow a similar pattern of sixteenth-note groups and bass notes, with some variations in the top staff's notes and rests.

129

Measures 129-135 of a piano score. The top staff uses a treble clef and the bottom staff uses a bass clef. The music consists of seven measures. Measure 129 starts with a sixteenth-note pattern on the top staff, followed by a bass note on the bottom staff. Measures 130-135 follow a similar pattern of sixteenth-note groups and bass notes, with some variations in the top staff's notes and rests. A rehearsal mark "2°/III" is placed above the bass staff in measure 130.