Euler's *Gradus Suavitas:* counting prime factors

In 1739, the mathematician Leonhard Euler published *Tentamen novae theoriae musicae,* a treatise in which he proposes his approach to measuring rationally expressed sounds.

Euler empirically defines a "degree of pleasantness" associated with any number, based on its prime factors. A prime number p is assigned degree p. For a composite number pq the degree is calculated by $p + q - 1 = 1 + (p - 1) + (q - 1)$. 1 is assigned degree I, 2 degree II, 3 and 4 degree III, 5 degree V, etc.

To calculate the degree of intervals and chords, Euler takes the least common multiple (or lowest common partial) of the pitches, and applies his equation to this number.

Euler's formula

For a number, expressed (uniquely) as a product of primes:

$$GS\left(\prod_i {p_i}^{k_i}\right) = 1 + \sum_i k_i(p_i - 1)$$