Harmonic Radius measures the partials' geometric mean

Given a set S of n pitches expressed as partials $\{P°_1, \dots, P°_n\}$:

$$Harmonic\ Radius\ (S) = \sqrt[n]{\prod_i P_i}$$

Log Radius measures the partials' arithmetic mean

Given a set S of n pitches expressed as partials $\{P°_1, \ldots, P°_n\}$:

$$Harmonic\ Radius\ (S) = \sqrt[n]{\prod_i P_i}$$

$$Log_2\ Radius\ (S) = \log_2 \sqrt[n]{\prod_i P_i} = \frac{1}{n} \sum_i \log_2 P_i\ .$$

Odd Radius

Define the odd partial pitch class of a natural number partial $P°$:

$$div2(P)$$

$$Odd\ Radius\ (P) = Harmonic\ Radius\ \big(div2(P)\big).$$

Odd radius is especially useful in situations where pitches are “octave-equivalent”, recurring in various octaves: e.g., in common scales and gamuts; in chord inversions.

Prime Limit Radius

Define the greatest prime factor for any natural number:

$$gpf(P), P \geq 2; \quad gpf(1) = 1$$

$$Prime\ Limit\ Radius\ (P) = Harmonic\ Radius\ (\{P, gpf(P)\}).$$

Prime Limit Radius captures the difference introduced by higher primes:
7:8 has a prime limit radius value of 6.06 while 8:9 has 4.56.