A LOOK AT HARMONY BY NUMBERS:
Notation and Composition with Respect to Rational Intonation

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Tempo #316 (April 2026)

I am a violinist by training: my lessons with Dorothy Pearce began in 1970 in Kitchener-Waterloo, Canada. Soon after, I was also writing little pieces of my own. In my lessons and earliest compositions, violin and piano went hand-in-hand. My teacher played notes, scales, and broken chords from the keyboard. I learned where to place my fingers on the violin by applying the most basic and difficult intonational tool: tuning sounds in *unison* with another instrument.

The colourful variety of sounding near-unisons awakens an embodied awareness of intonation: exact purity, shining resonances and consonance; beating; clangourous, joyous inharmonicity and spicy dissonances; eventually, the sound becoming rough and sour. So many colours and shadings in such a subtle phenomenon! Building analogies to sight, touch, taste and smell, intonation is, for me, a supremely multi-modal sensory experience, a way to identify and distinguish sounds in relation to each other and, at the same time, to shape and colour sounds in time.

String players eventually encounter double stops: perhaps when learning to adjust the open strings to each other, or possibly in a book of studies where two notes are to be played at once. We are invited to tune, but *not* by matching unisons. Musicians and singers who are free to intonate their sounds do so by learning to recognise special relationships – consonant intervals – which, acoustically speaking, are ratios of frequencies that can be neatly expressed as proportions.

Every pitched sound is a repeating pattern of vibration. Whole number frequency ratios between pitches produce a composite repeating pattern, a fast polyrhythm. An interval, like a rhythmic beat, has its own unique 'groove' or *periodic signature.* A pitched sound is also a unique spectrum, a specific collection of harmonic partials. An interval may be distinguished by the (near) correlations of these partials, producing similar phenomena to those heard when tuning a unison. Finally, the superposition of frequencies vibrating in a shared medium (the air, an eardrum, the fluid filling the inner ear) creates various combination tones.

This acoustical interaction of tones generates psychoacoustic responses. Real and virtual sounds are combined into layered sensations. Intervals in which lower partials align – frequency ratios of relatively small whole numbers – have a stable waveform. They are acoustic manifestations of rational frequency relationships and are thus part of a way of tuning called rational or just intonation (JI). Such intervals may be recognised and exactly tuned by ear, adjusting the pitches to minimise phase drift and beating; I call them 'tuneable' intervals. Early on in my involvement with JI, I published a list of intervals within an 8-octave range that, to my ears, were tuneable.[1] But before I could do so, I needed a notation.

[1] Marc Sabat, '23-LIMIT TUNEABLE INTERVALS below "A4"', masa.plainsound.org/pdfs/TIab.pdf (26 February 2026).

As a violinist, I experienced how tuning in different contexts leads to pitches different from the 12 tempered notes available on a piano. It seemed strange to me that music notation did not distinguish these 'microtonal' intonations. I became curious to *compose* such sounds, to make harmonies and counterpoints with them, to be able to write and read them in a score.

Gradually, I learned about the music and theoretical writings of earlier microtonalists like Harry Partch, Elsie Hamilton, Ben Johnston, La Monte Young, Pauline Oliveros, and James Tenney, all pioneers of JI composition. Their work relied on notating fractions, cents deviations from Equal Temperament, tablature, or demanding a practice based in aural tradition. Upon moving to Berlin in 1999, I met Wolfgang von Schweinitz, and over several years we developed the Helmholtz-Ellis JI Pitch Notation (HEJI), which has been adopted by many contemporary composers working with JI sounds (Catherine Lamb, Thomas Nicholson, Juhani Nuorvala, among others). The most recent version of the notation, along with fonts for music notation and text, was released in 2020 in collaboration with Thomas Nicholson.[2]

Rational intervals present me with a multitude of new sounds I have not yet experienced, which I can perceive and musically explore. Although I can appreciate the convenience of tempering tones to approximate a variety of different potential identities with a single pitch, and it is possible to sense the equality and symmetry of an equal division system, I have not yet discovered any harmonic sonority or progression that depends *exclusively* upon such a construction. My experience of harmonic sound is fundamentally rooted in the psychoacoustics of the harmonic partial spectrum, and working with rational intervals is essential for me to understand and compose harmony.

The HEJI Notation

In some ways Wolfgang von Schweinitz and I could not be more different. Through his early music, Wolfgang became associated with a new Romanticism that arose in Germany in the 1970s, while I see myself more closely connected to North American experimentalism. Yet, as soon as we met, it became clear we shared an overwhelming conviction: we wanted to compose music exploring the harmonic sounds of JI and to make this music accessible to instrumentalists.

In our first meetings, we talked about a wish to more fluidly and intuitively write down, hear and play JI pitches. We did not want to limit ourselves to a fixed scale but rather to be able to modulate freely, using staff notation and visualising ratios. We wanted to be able to transpose intervals and entire passages of music easily. We wanted a microtonal notation that was clear, legible in larger scores and readable by musicians.

Hermann von Helmholtz had suggested tempering the difference between eight pure fifths (2:3 ratios) and a pure major third (4:5 ratio) to make an enharmonic equivalence. For example, the syntonic comma-lowered E a just major third above C could then also be tuned in a chain of minimally tempered fifths.

[2] Marc Sabat, 'The Helmholtz-Ellis JI Pitch Notation (HEJI) | 2020', heji.plainsound.org (26 February 2026).

♮↓E

♮↓E=♭F – ♭C – ♭G – ♭D – ♭A – ♭E – ♭B – ♮F – ♮C /

Figure 1: Chain of fifths produces a third in Helmholtz Temperament. Each fifth is made smaller by about 1/4 cent.

He indicated syntonic comma alterations of the Pythagorean series of notes and thought this would suffice as a basis for JI music: Helmholtz believed that higher primes were, with perhaps a few exceptions, too discordant for general use in music.

Wolfgang proposed that HEJI notation take flats, naturals, and sharps to represent a Pythagorean series of untempered fifths and to write syntonic commas as arrows attached to the traditional accidentals: 5-limit JI would look like ordinary music with explicitly notated commas. Transposition to different keys was clear and easy. The 'circle of fifths' would simply become an endless spiral.

In his English translation of Helmholtz, Alexander Ellis had also introduced cents as a practical notation for comparing microtonal variations of tuning; over time, his method has been universally adopted as a convenient representation of any microtonal scale.[3] Tenney adopted Ellis' approach in his microtonal compositions, combining cents deviations with frequency ratios. This hybrid approach may be readily integrated with HEJI accidentals since it is easy to see which chromatic note is being modified at any given time. For woodwind players, cents are an essential tool to train ears and develop playing techniques, likely requiring them to find alternate fingerings.

Harry Partch tried various notations but settled on a combination of instrument-specific tablatures, conventional pitches and fractions to describe notes used in his music as ratios derived from one central generating tone. For the most part, he used a fixed scale with 43 JI tones per octave, which introduced the higher primes 7 and 11, an approach Ben Johnston refers to as 'extended just intonation'.

Johnston developed a staff notation using accidentals for various primes, but his system does not separate the Pythagorean series and the syntonic commas as clearly as Helmholtz' approach.[4] The accidentals combine tertial and quintal ratios, requiring constant re-evaluation of commas after any diatonic transposition. This can, however, become counterintuitive. Wolfgang and I chose to define symbols for each additional prime in terms of nearby Pythagorean pitches.[5] Any JI ratio may be notated in this way, by combining the higher prime symbols with the appropriate combination of syntonic arrows and Pythagorean flats/naturals/sharps. Transposition by a diatonic interval means simply changing the letter name and possibly altering a natural to a flat or sharp.

[3] The *scala* file format, developed by Manuel Op de Coul, uses both ratios and cents, and has become one of the most popular standards for retuning computer music instruments: Manuel Op de Coul, 'Scala scale file format' https://www.huygens-fokker.org/scala/scl_format.html (26 February 2026).

[4] Marc Sabat, 'On Ben Johnston's Notation and the Performance Practice of Extended Just Intonation' https://masa.plainsound.org/pdfs/EJItext.pdf (26 February 2026).

[5] The first version of HEJI, from the early 2000s, defined some higher primes in terms of lower limits. As more experience was gathered from various composers' practices, it became clear that a strictly Pythagorean, prime-by-prime approach was needed, and therefore in 2020 HEJI was revised accordingly.

In the current standard, revised in 2020, we define symbols for primes up to 47, graphic signs that provide a visual and musical complement to tuning by numbers using fractions, cent deviations and frequencies.

Ricercar

One of the earliest experiments Wolfgang and I undertook together in the early 2000s was making a JI tuning of Johann Sebastian Bach's *Ricercar a 3* from *Musikalisches Opfer*.[6] With HEJI notation as a tool, decisions could be made in advance – an intonation composed. Bach's chromatic but tonally structured subject and three-part counterpoint setting was an ideal test case.

We wanted to hear how it would sound to consistently use JI in complex, chromatic, modulating counterpoint. We decided to specify a contextually determined tuning for every note in the score, maintaining tonal logic, without using any irrational or tempered pitch relationships, to find out how it might compare to common practice, and what principles might emerge from making these decisions.

Musicians are always tuning while playing music made with tones; choices are being made about how intervals and chords should sound. A single, fixed intonation may be assigned to each written note: a keyboard temperament, for example. Alternatively, notes may be tuned flexibly on a case-by-case basis, in real time, presenting microtonal variations based on an interpretation of the interplay of simultaneous and successive tones – on chords and melodies. As notes flow between different tonal relationships, the partials that best represent those relationships change and intonation must be adjusted accordingly.

To test these ideas, we devised a workflow using the notation program Finale and its MIDI output capacity. A sampled software instrument, the Unity DS-1 Reed Organ, was tuned to a 24-note Pythagorean series using two 12-note custom scales on two different staves and channels. All notes were tuned in untempered perfect fifths, so these two scales were a Pythagorean comma apart.

𝄫B – ♭F – ♭C – ♭G – ♭D – ♭A – ♭E – ♭B – ♮F – ♮C – ♮G – ♮D –
– ♮A – ♮E – ♮B – ♯F – ♯C – ♯G – ♯D – ♯A – ♯E – ♯B – 𝄪F – 𝄪C

Figure 2: Two 12-note scales a Pythagorean comma apart forming a 24-note series.

Further pairs of staves in our playback score could then be assigned for each new HEJI accidental combination. Thus, one stave-pair could play Pythagorean pitches lowered by a syntonic comma, another could play tones lowered by a septimal comma, and so on. Using cut-and-paste, notes could be assigned different tunings and compared.

[6] For a discussion of the working process with examples of how decisions were made see my 2003 article: Marc Sabat, 'Johann Sebastian Bach RICERCAR Musikalisches Opfer 1 INTONATION by Marc Sabat and Wolfgang von Schweinitz (2001) an experimental application of Extended Rational Tuning' https://masa.plainsound.org/pdfs/ricercar-text.pdf (26 February 2026).

The working process involved writing down notes on paper, deciding on options, copying them into the correct staves, and comparing the playback. The clear and rich sustained spectrum of the reed organ allowed us to easily hear the different intervals and qualities of a specific JI tuning (beating, combination tones, periodic signature). We could study how JI sound combinations make counterpoint, how intonation affects the emergence of tonal structures over time.

The first five notes of Bach's subject sketch a tonal context: an arpeggiated C minor triad in root position, with two neighbouring diatonic semitones above and below, suggesting the subdominant and dominant chords. This is followed by a chromatic descent, and the subject concludes with a tonal cadence that sounds the subdominant and dominant chord tones that were missing in the first three bars (F and D respectively).

We began by mapping the 11 pitch classes of Bach's subject in 5-limit or Ptolemaic tuning, which connects perfect fifths and major thirds into an interlocking tonal lattice of triads.[7] This is possible because the triads consist of a major third (4:5, upward to the right) and a minor third (5:6, downward to the right), which combine to make a perfect fifth (4:6 or, in lowest terms, 2:3, directly to the right). In the diagram below, arrows indicate alteration of Pythagorean notes by syntonic commas (using HEJI notation). The missing 12th pitch class B-flat, completing a minor triad rooted on G, emerges as the first note of the countersubject, in bar 10.

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        (♮↓A) – ♮↓E – ♮↓B – ♯↓F
               /      /      /
     ♮F – ♮C – ♮G – ♮D – ♮A
      /     /     /
↑♭D – ↑♭A – ↑♭E
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Figure 3: 5-limit tonal lattice of the Bach *Ricercar*, bars 1–9, including two A's one comma apart

In historical quarter-comma Meantone temperament, perfect fifths are tuned 1/4 of a syntonic comma narrower so that a series of four perfect fifths produces a harmonic major third 4:5. In the lattice above, for example, Meantone C would be tuned 1/4-comma lower, G 1/2-comma lower, and so on, so that, finally, A would be tuned a comma lower, a major third above F. In other words, the two dimensions, fifths and thirds, may be collapsed into a single row. In JI, however, the fifths remain untempered, so the two A's remain a comma apart. Nevertheless, both systems share a common interval: the 4:5 major third.

[7] The tone lattice was pioneered by Leonhard Euler to visualise consonant major and minor triad relationships based on frequency ratios 2:3 (perfect fifth) and 4:5 (major third). Diatonic seven note modes may be represented in the lattice so that some of their triads are tuned in small number proportions. For the tonal major and minor modes, this is usually done by taking three major or minor triads rooted on a central pitch (the tonic), a fifth above it (the dominant), and a fifth below it (the subdominant). This method produces one 'wolf fifth', which generally means that one (or more) of the notes in the scale need to have several possible tunings one syntonic comma apart. In the *Ricercar* lattice above, A is such a note; it may be tuned either as 9°/G or a comma lower as 5°/F. Modulating between tonal centres not only changes notes chromatically (by sharp or flat accidentals), but also enharmonically (by commas).

Major triads are built with the major third below the minor third, while minor triads place the major third above. This structural difference has psychoacoustical implications, which JI tuning accentuates by highlighting our perception of difference tones. A triad has three constituent intervals, and therefore three first order difference tones. The major triad combines partials 4:5:6, producing a difference tone chord 1:1:2, sounding a fundamental that reinforces the root of the chord by doubling the same pitch class in lower octaves.

By reversing the thirds, the minor chord is positioned further away from its fundamental. It combines the partials 10:12:15 and its difference tone chord, 2:3:5, is itself a major triad, sounding a fundamental not present in the minor triad itself. It introduces a tension between the root of the minor chord and the implied harmonic fundamental, a virtual sound which lies two octaves and a major third below the root.

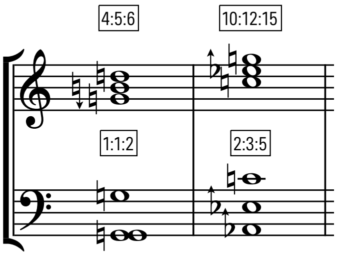

Figure 4: 5-limit major and minor triads in root position with their first-order difference tones.

Returning to the tuning of the *Ricercar* subject in 5-limit JI, the leading tone B is tuned as partial identity 5°/G, where G, the root of the dominant major triad, is partial identity 3°/C.[8] Similarly, A-flat is tuned as part of the subdominant minor triad where the notes are, respectively, partial identities 5°–3°–15°/D-flat, as discussed above.

♮F – ♭A – ♮C =
5° – 3° – 15° / ♭D

Figure 5: Subdominant minor triad.

In the lattice (Fig. 1), notice that any tones placed to the right or above indicate *overtonal* relationships. Thus, comma-raised D-flat is the fundamental of all the pitches mapped out.

♮B = 225° / ♭D
♮G = 45° / ♭D
♭D – ♭A – ♭E = 9° / ♭D

Figure 6: Lattice fundamental is comma-raised D-flat.

[8] The notation 1°, 3°, 5°, etc. refers to harmonic partial 'identities' (characteristic overtonal intervals, adapting a terminology introduced by Partch). The odd numbers represent pitch classes derived from a fundamental. Thus the 5° identity is the major third, 7° the natural seventh, 11° the quartertone-raised fourth, etc. Expressing a collection of pitches as identities of a single fundamental becomes a key concept in the development of harmonic radius.

Bach's theme continues with a chromatic scale, descending from G down to B before cadencing by rearticulating the triadic tonality of C minor. If the tones are derived from our 5-limit lattice, this descending scale is melodically somewhat uneven, with three differently sized descending semitones, and it is tuned over a tonally distant common fundamental, D-flat. In Figure 7, the tuning A is set to 0 cents, so C is −6 cents.

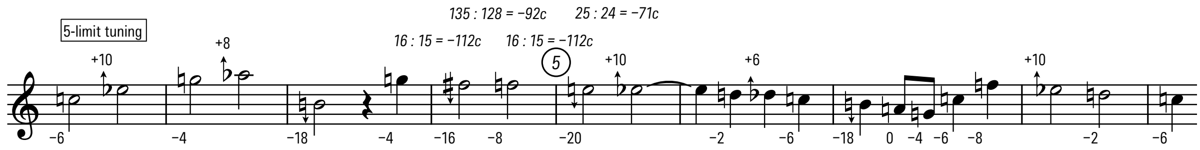

Figure 7: 5-limit tuning of the Bach *Ricercar*, bars 1–9

Another JI possibility is to tune the chromatic scale from successively descending harmonic partials, as in Figure 4. This approach introduces higher prime identities 7°, 17°, and 19°, using common tones to connect the implied harmonic fundamentals, C and G, which also function to reinforce the tonality. Note that 17° and 19° happen to fall very close to 12edo[9].

Prime partial 7°, on the other hand, deviates the most from 12-tone well-tempered tunings, although it is a familiar sound in quarter-comma Meantone. In the standard historical keyboard temperament it is heard as an augmented sixth and usually found between B-flat–G-sharp or E-flat–C-sharp. Examples of septimal harmonic usage, although rare, do occur in music by Michelangelo Rossi and Girolamo Frescobaldi, among others.[10] This partial is most commonly sounded by musicians when tuning the diminished triad (5:6:7) or diminished fifth interval (5:7). This latter sound is an enharmonic near equivalent to the slightly rougher 5-limit tritone 32:45. In our tuning of the *Ricercar*, we play with this intonational similarity in bars 10–12 (see Figure 8 below). Here the opening subject is answered in the middle voice, beginning on G. Its first four tones present a slight tonal alteration of the theme to reinforce the key of C minor by transposing the first note of bar 2 down a fifth (to C) instead of a fourth. The missing D is instead heard a bar later, where it initiates the chromatic descent.

[9] 12-tone equal tempered tuning; 'edo' means 'equal divisions of an octave'.

[10] Thomas Ciszak, 'Frescobaldi and the Natural Seventh', presented at the Winter Musik, Akademie der Künste Berlin, 2022.

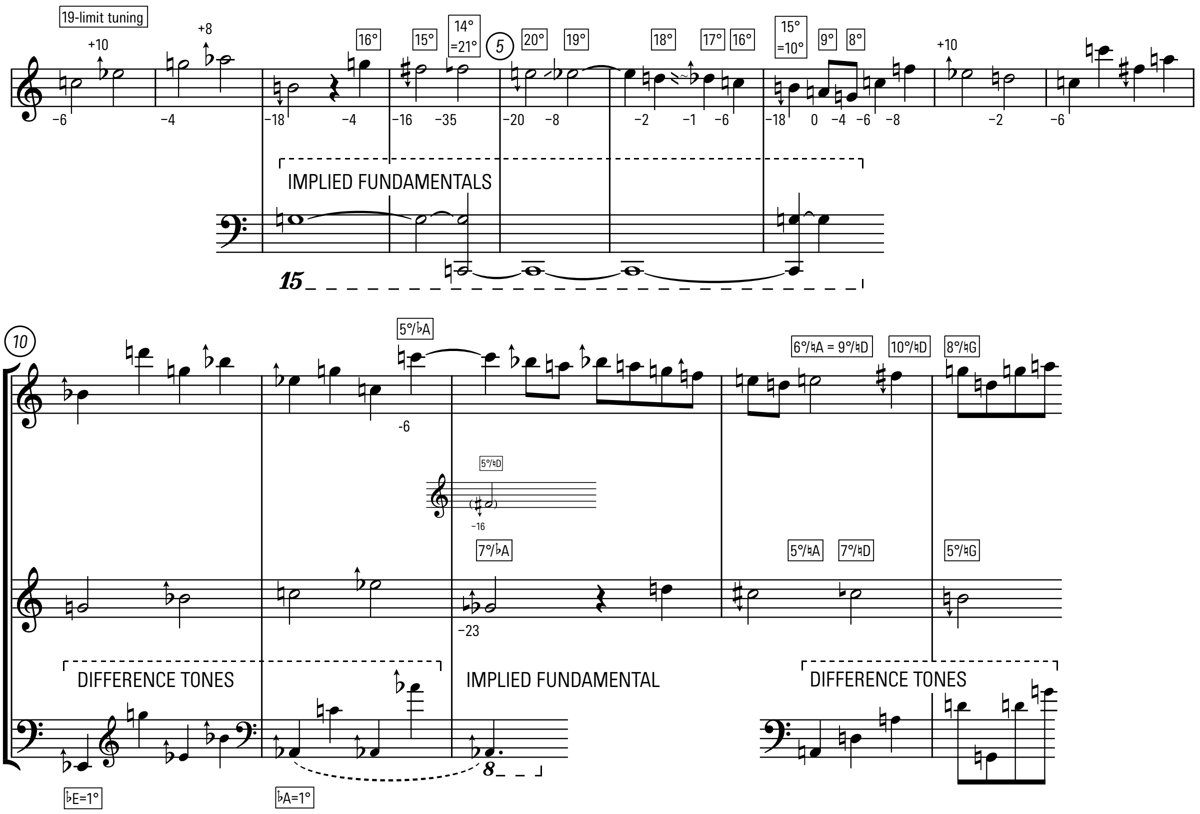

Figure 8: 19-limit tuning of the Bach *Ricercar*, bars 1–14

Meanwhile, the countersubject presented in the upper voice completes minor triads on G in bar 10 and C in bar 11. In bar 12, the upper voice is tied over, sounding C, while the middle voice leaps down, suggesting a secondary dominant rooted on D. In terms of tonal logic, the 'correct' tuning would be F-sharp = 5°/D. The C in the upper voice changes function from root of the triad in bar 11 to seventh in bar 12, before 'resolving' melodically downward in bar 12.

In some circumstances, it might be possible and appealing to introduce a harmonic seventh tuning, even anticipating a seventh function, particularly when the seventh resolves down to the major third as in bars 13–14. Further examples of this may be found in bars 75–77. However, in bar 12, Bach allows the resolution of dissonance to be implicit in the downward movement of the upper voice, while the sustained half-note F-sharp suspends it acoustically. We decided to underscore this moment with a small harmonic experiment. As noted above, the untuneable interval 32:45 (590 cents) is enharmonically close to the simpler, smoother, tuneable 5:7 (582.5 cents).[11] Thus, F-sharp may be tuned slightly lower, by about 1/3 comma, becoming a septimally-lowered comma-raised G-flat.

In the context of the preceding bar, this enharmonic retuning allows all the notes and difference tones to be expressed as harmonic partials of a low comma-raised A-flat. It briefly reinforces the psychoacoustically implied harmonic fundamentals, which, in a 5-limit tuning of minor tonality, are

[11] Note that the C is voiced above the comma-lowered F-sharp, plus an extra octave, so the ratios are inverted as well: 45:128 vs. 7:20, exaggerating the difference in tuneability and perceived concordance.

different from the tonal roots, as discussed above. The G-flat respects Bach's treatment of voice leading (by unresolved dissonance) and heightens it by eliding the 'correct' major third of the secondary dominant function (F-sharp) on the fourth eighth note of bar 12.

In bars 13–14, septimal tuning reappears in a two-voice context, this time in a way that strengthens the tonal root movement with difference tones. The middle voice follows the descending chromatic tuning previously established in bar 4.

♮D – ♯↓C – ⌞C – ♮↓B
16° – 15° – 14° / ♮D = 21° – 20° / ♮G

Figure 9: Chromatic descent in bars 13–14.

Note that the septimal C is not a dissonance to its root note (14° = 7°/D), but it *becomes* a dissonance to the root of resolution (21° = subminor fourth over G). The extended JI setting of the counterpoint also creates a novel transformation in the interplay of consonant and dissonant sounds. In an historical tuning, C–E would be a consonant major third. Here, the interval is a very large, dissonant, but tuneable 'supermajor' third with ratio 7:9, producing a difference tone 2 = D, followed by a slightly more dissonant tritone (7:10), resolving to a relatively consonant minor sixth (5:8) with implied fundamental G.

Partials 19° and 17° equally divide the whole tones 10:9 (minor tone) and 9:8 (major tone) respectively. Thus, in combination with neighbouring partials they can produce a smooth melodic sequence of even semitones, while at the same time introducing new harmonic possibilities. For example, the step from 20° to 19° lowers the major third (4:5 = 16:20) to a minor third (16:19), which has a difference tone 3°. The resulting chord, 3:16:19 is a tonal minor triad in which the root (16°) and implied fundamental (1°) share the same pitch-class, offering an alternative to the 5-limit chord 10:12:15. Depending on the voicing chosen, the 19-limit minor chord can, despite the higher prime identity, sound very consonant; as noted above, it also happens to lie very close to 12edo.

Similarly, the smooth diminished triad, 5:6:7, may be extended into a half-diminished chord by adding partial identity 9°, which is the same pitch class as 18°. 17° is a semitone (99 cents, also close to 12edo) lower and therefore makes a fully diminished chord 10:12:14:17: Here the pairwise first order difference tones are 2, 2, 3, 4, 5, and 7, reinforcing the chord tones, and adding an implied fundamental that reinforces tonal function, since the diminished chord may be thought of as a dominant flat 9 with missing root from the harmonic minor scale.

In our *Ricercar* tuning, all of these aspects of 19-limit tuning come into play, and the three part contrapuntal texture is especially suitable to give these sound combinations clarity and context. Tuning the sounds in JI also produces very clear differences in the time-based listening experience when compared to a well-tempered or fixed system tuning.

Harmonies in Bach's music do not move in blocks but are rather gradually revealed, note by note. Often a sustained tone is presented in one voice and then moving notes in another voice reveal or transform the harmonic function(s) of the held note. In a tempered version, ambiguities of tuning force the ear to draw on previous listening experiences and context to determine how to hear the harmonic relations, to deduce what intervals are implied in spite of the mistuning.

In a JI version, the intervals are explicitly sounded and by hearing and recognising them it is possible for a listener to psychoacoustically derive potential harmonic relations. In the case of the major chord and its inversions, this is completely unambiguous. But in the case of minor triads, dominant 7th chords, diminished chords, or chromatic alterations, various harmonically valid and acoustically salient options exist. Some of these also sound unfamiliar or distinctively strange. In particular, if a higher partial is presented first, without lower partials giving context, it can seem out of place or out of tune. Some choices lead to melodically strange voice leading, even if the harmonic sounds are themselves consonant. Testing how this works requires empirical research, and especially with real instruments and musicians in a real acoustic space.

The tuning of notes in JI fulfils or reveals harmonic relations to come, and this can either allow for a more relaxed listening experience or create an unusual anticipatory dissonance. In both cases, the temporal experience of harmonic flow is transformed by the JI setting. This potential for perceiving tuning as out of place is mitigated by simultaneously sounding harmonic sounds and becomes more obviously uneven or microtonal in a melodic or solo context.

Moving Towards Harmonic Radius

Looking back at some of my recent compositions, I find myself composing recurring, distinctive small-number four-note chords used as characteristic 'consonances' of the higher prime partial identities 7°, 11°, 13°, 17°, 19° etc. For example, I use 6:7:8:9, 8:9:11:12, and 10:12:13:15, which (respectively) are the smallest-number tetrads within a perfect fifth introducing primes 7, 11, and 13. For even higher primes, I often like the sound of combination-tone reinforcement chords, for example 8:9:17 and 3:16:19.[12] In my *Fleeting Flight Sleeping Woke* (2021) for 1/6-tone Haba Harmonium and ensemble, I use an extension of this technique to build 'summation-tone trees', larger chords that successively reduce higher primes as summation tones of lower primes, extending the process until 3-limit pitches are obtained.

These chords highlight the sonority of a prime by embedding it within an tertial structure.[13] I work freely with different voicings of these tones, treating them as potential pitch classes, while keeping in mind how octave transposition of partials changes the combination tones and transforms the chord's relative harmonicity and concordance.

[12] As used in the music of Horatiu Radulescu and Claude Vivier, among many others.

[13] This approach was inspired by how 7° is combined only with 3° in the tuning of La Monte Young's *The Well-Tuned Piano* (1964–1987).

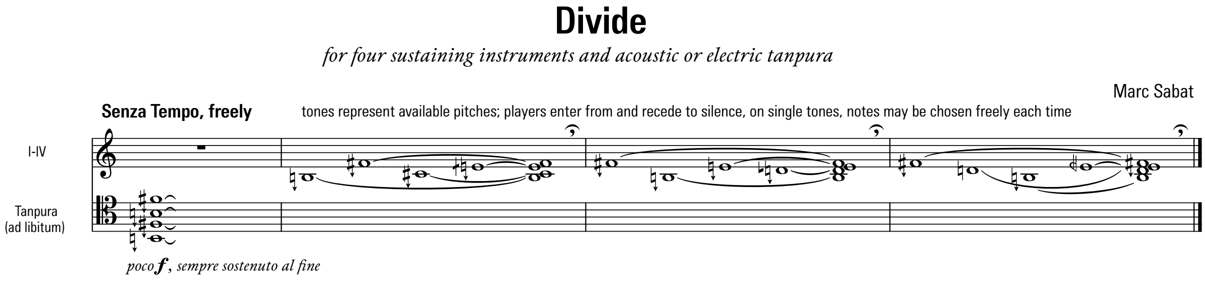

Figure 10: *Divide* opening, with three close position higher prime limit tetrads

Underlying all of these examples is a wish to find salient, typical and tuneable chords that use higher primes, a kind of analogue to the tuneable intervals, applicable to JI aggregates and harmonic changes. As intervals embrace higher numbers, their sounds become less characteristic, less evidently periodic. Dyadic counterpoint with 11°, 13° and higher primes often requires ever wider intervals to obtain sonorous fusion. Having more voices mitigates this, although the sonorities remain quite intense, as in Figure 11.

Figure 11: *Gioseffo Zarlino* opening, 13-limit counterpoint (bass flute, viola, cello)

In the bass flute part of *Gioseffo Zarlino* (2015), 13° interacts contrapuntally with 5-limit two voice counterpoint between viola and cello, building on different combinations of the characteristic chord 10:12:13:15. The piece repeats the same cycle many times, with changing instrumentation, so the flute emerges gradually. I especially like how the 13° begins to darken a previously idyllic texture, giving it a smoky and funky depth, which our ears first question then very gradually come to accept as harmonic and integrated, and finally even miss when it recedes and disappears.[14]

Elsewhere I have correlated tuneability as offering a new definition of 'consonance' based on perceptibility, a way to musically integrate higher regions of the harmonic partial row and their fine microtonal differences in a way that is, to my ears, potentially beautiful. Following this line of curiosity underlies the second work explored in this paper: a new and ongoing composition called *The Tree*, named partly in homage to Tenney's final work *Arbor Vitae*.

The Tree

When I first imagined composing with JI sounds, back in the early 1990s, as I was learning about frequency ratios by reading Partch's *Genesis of a Music*, I naively envisioned something akin to David Attenborough's wonderful nature documentaries: a colourful world of alien-like plants and species inhabiting a wild, interwoven ecosystem previously unknown to me. I believed there would be a self-evident, natural flow in the expanded harmonic space. Of course, such an environment comes with its own natural laws of attraction and repulsion, and as I began to learn the sounds of higher primes and to make first compositional attempts with these relations, I was surprised and perhaps a little disappointed by how much resistance, friction and harshness I experienced. Unlike the complex timbral worlds of noise and sound textures, in which we let go of harmony, there is something intensely challenging about microtonal JI. Listeners welcoming experimentation with sounds in New Music might still find these tuned, distant partials austere and harsh.

I was eager to explore higher primes; I wanted to find a way to move freely within a wider harmonic space, into new harmonic regions. But working with tuneable dyads only opened the door so far beyond the 7-limit for me. Using the characteristic chords was a first step, giving me something like the stable extended harmonies in jazz standards, chords that were not only JI variations of major, minor, diminished, augmented triads, sevenths and so on, but new harmonic colours.

The Tree is an ongoing project. To date there are eight pieces in the cycle, some evolving, some having been performed by various ensembles. Because it is still in progress (and I hope it will continue to grow over the coming years), I will just sketch out a few observations.

The piece began with a trilogy initially called *Partial Branches*, written for and premiered by Harmonic Space Orchestra. Its first section, *SKY* (see Fig. 14), begins by introducing partials 2°, 5°, 15°, 25° of a low C, which then stops sounding, leaving a branch over 5°, the syntonic-comma-lowered E. The 25° (5° of E) moves melodically stepwise upward through 26° and 27°, sounding the 3° branch over G, then up to 28°, sounding the 7° branch over septimally-lowered B-flat. The music continues in the middle register, sounding some characteristic chords of 7°, 11°, 13° and the 5-limit. The final chords of the piece explore differences of harmonicity between four note structures in the region 10:11:12:13:14:15.

[14] A recording can be found here: Marc Sabat and Harmonic Space Orchestra, *Gioseffo Zarlino* (Sacred Realism, 2020), CD.

SKY is followed by *EARTH* and then is reprised and slightly extended an octave lower in *GROUND*. The arc downward in register highlights the edges of dissonance in 3- and 5-limit low register sounds and also opens up the upper register to higher odd identities (17°, 19°, 23°, 35°), moving towards the sounds that other sections of *The Tree* will highlight.

Partial Branches

Marc Sabat

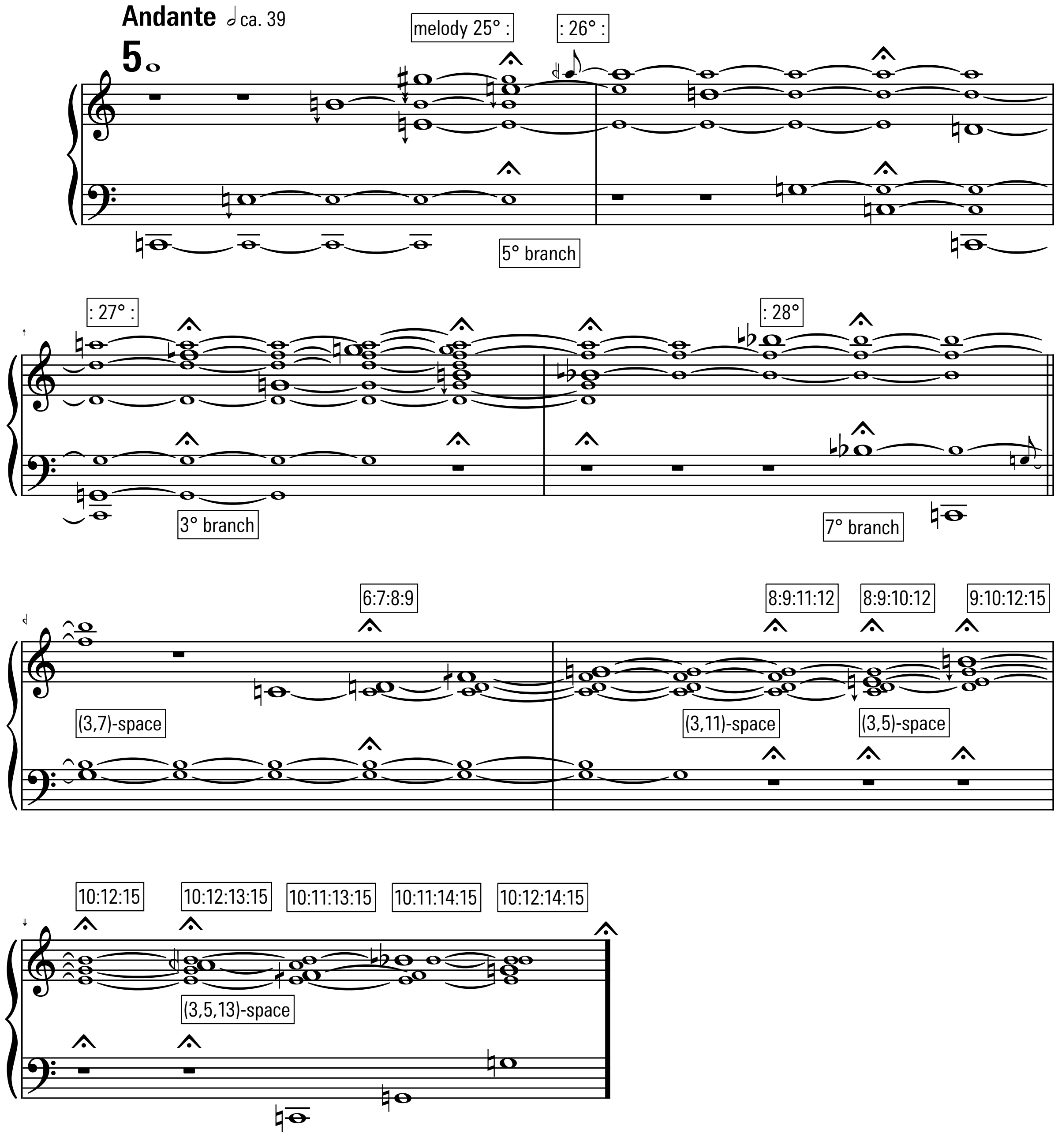

Figure 12: *The Tree (Partial Branches), SKY,* particell

The next section composed was *47 CLOUDS*, which was my second time writing sounds with partial 47°. In *Fleeting flight sleeping woke* (2021), for each of the high primes available on the Haba 1/6 tone harmonium I had made collections of chords I called 'summation-tone trees', described above. I began by reinforcing the high prime partial with two lower partials that add up to it, working my way down the prime limits until reaching tertial notes. This gave me sequences of different chords with 47° as a common tone summation tone, creating melodies of partials in lower octaves, which I found especially appealing with a quick rhythmic profile.

Figure 13: *Fleeting flight sleeping woke,* summation tone trees below a sustained 47°

Revisiting 47°, I wondered if I could find four-note chords in closer position that I liked as well as the large high-fusion but rather dissonant many-toned trees. I began by looking at the difference tones of nearby partials, beginning with 2:45:47, which shaped the first two chords with 47°, 32:36:45:47 and 30:36:45:47.

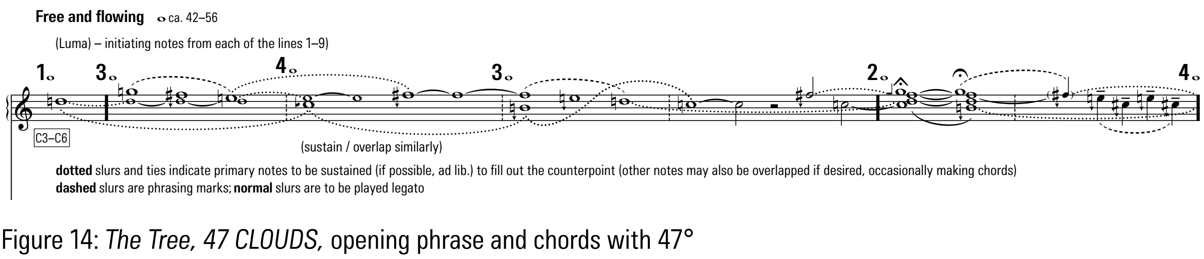

Figure 14: *The Tree, 47 CLOUDS,* opening phrase and chords with 47°

This experimentation was a direct outgrowth of my work with the Lumatone instrument. Although the instrument is conceived for isomorphic layouts of equal division tunings up to 55edo, it can also be applied to JI pitch sets like Partch's 43-tone scale or Wilson's combination product sets. As I was getting to know the instrument, I worked during 2022–23 on developing an open source app, the PLAINSOUND Hexatone, which interfaces with a Lumatone controller and offers a touchscreen hexagonal keyboard experience featuring various built in scales and sounds.[15]

I became interested in using the Lumatone as the core of a contemporary 'continuo' formation, so I sought an appropriate layout of pitches to work with. I applied various odd partial pitch class sets from my harmonic radius research[16], eventually settling on an odd limit 255, prime limit 47 set of 81 pitch classes. This pitch set is large enough to permit a wide range of possible modulations, and every note may be combined in an harmonic chord of at least three different pitch classes. In particular, the 12 chromatic pitch classes of 12edo are all very closely represented.

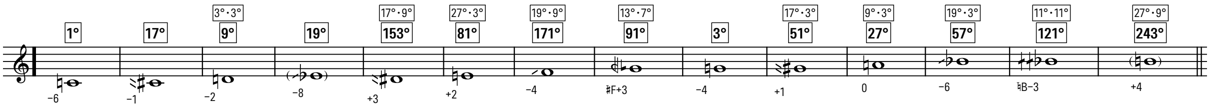

Figure 15: *The Tree,* 12 tone chromatic scale within the 81 odd partial pitch set.

One of the unique features of the Lumatone is the ability to control the visual appearance of each key by specifying its colours. To orient players in the forest of hexagonal keys and all the identities of my JI set. I gradually developed a way of colouring each of the primes and their combinations in a way that allows for fluent playing.

Working with the Lumatone, I began trying various summation tone chords, moving between sounds, adding partials by ear, sometimes transposing by octaves, finding melodies. In the 81-odd set, 47° is the highest prime. Its partial branch is just a 5-limit major triad, comprising its 1°, 3°, and 5° identities. The structure of this chord suggested the simple form of this miniature movement. The opening phrase recurs at bar 14 a perfect fourth lower, cadencing on the 3° identity of 47°. The music closes with a third variation of the phrase, ending a major sixth higher, on the 5° identity of 47°. As this little piece took shape, it suggested to me that *The Tree* ought to include a set of movements highlighting characteristic sound combinations of the higher primes, and I am continuing to imagine how these might be composed.

Shortly after finishing *47 CLOUDS*, I sketched out a descending microchromatic scale from middle ♮C4 down to ♮C3, comprising the entire set of 81 tones. I was curious about the relationships, the connections of partials between neighbouring pitches, and how they might be harmonised into a progression moving in relatively close position voicing. In early 2025 I had managed to find the first few chord changes. I had a meeting with my colleague Sebastian Dumitrescu, who is also working with Lumatone. I wanted to share the pitch set and its colour layout. I remember how intuitively he was able to navigate sound combinations at the keyboard. The fact that the set of notes, though

[15] *Plainsound Hexatone*, https://hexatone.plainsound.org (26 February 2026).

[16] Marc Sabat, 'Chords, Melodies: A Look at Harmony by Numbers; Part I: Using Harmonic Radius to Compare Rational Pitch Collections', *Živá hudba*, 14 (2023), pp. 56–85.

huge, was also somehow playable, inspired me to move forward and make a piece moving swiftly through the entire descent.

I began with the five or six chords I had composed and wrote some free music gradually moving through the sounds. This remains an unfinished pencil sketch but working on it I found my footing with the close position sounds. I returned to the descending progression, and over several weeks of intensive work, *FALL* was composed. Each bar is built on one of the 81-odd pitches. The descent begins on D4, falls through a perfect fifth to G3, leaps up an octave to G4, and descends to D. This cycle repeats, and, on the second time through, continues down to C. The one larger change of register in bar 50a where the bass leaps up an octave seemed necessary to shape the music on a larger timescale and to break the straight descent. The example below shows the Lumatone solo setting, including fingerings. The boxed odd numbers are the descending pitch classes expressed as partials over C. Note that the descending line is not always the lowest sounding voice.

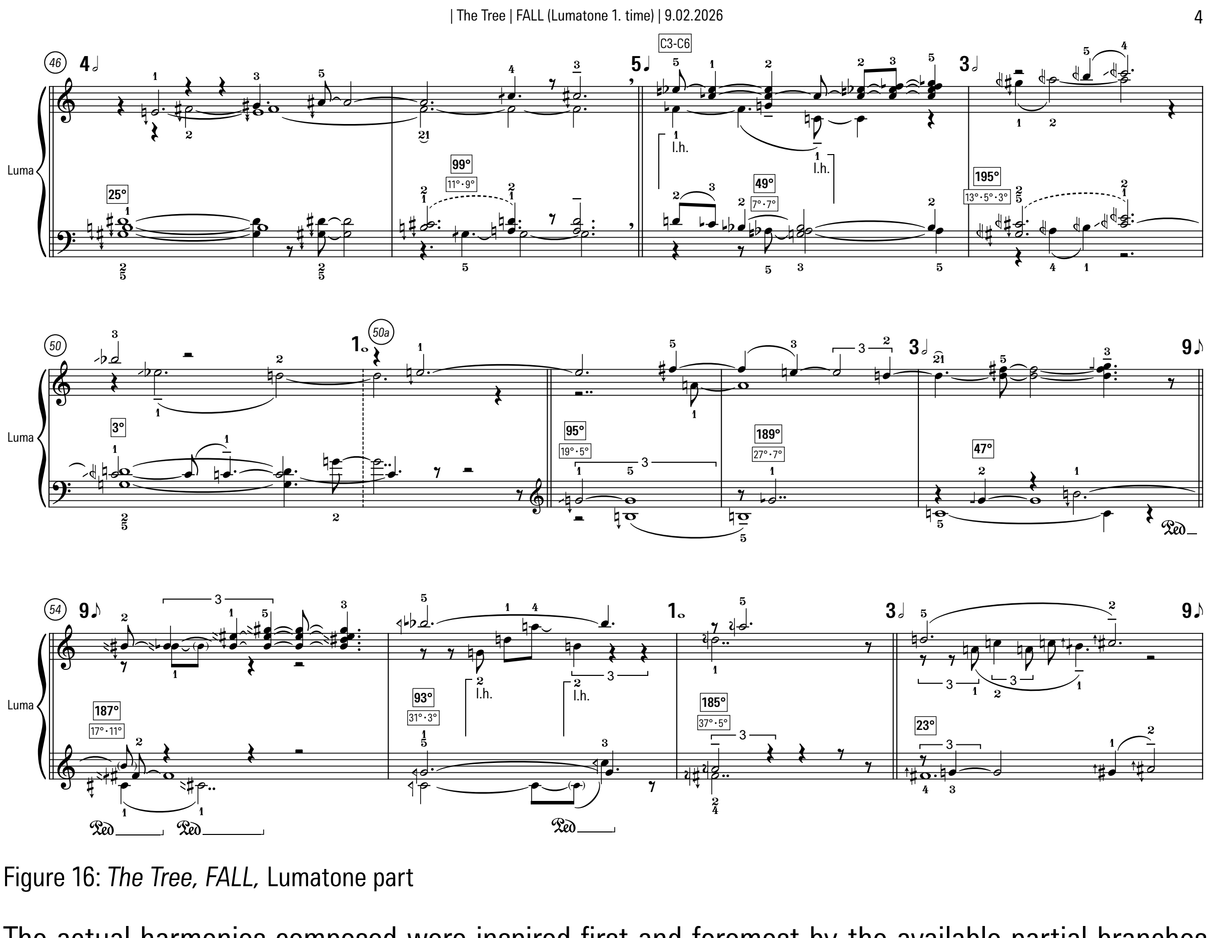

Figure 16: *The Tree, FALL,* Lumatone part

The actual harmonies composed were inspired first and foremost by the available partial branches and by multiple possible identities. For example, 35° can be a 5° or a 7° identity. In addition, I was searching for ways of combining higher primes and melodies that I found appealing. As small motives became associated with particular primes, I could follow their recurrences in the set by returning to previous material to create a memory form in the music.

To date I have made two settings of *FALL*. The first performance on 3. December 2025 featured Sebastian Dumitrescu on Lumatone and Cara Dawson on scordatura harp at KM28. A second performance featuring an arrangement for eight musicians (Lumatone, harp, flute, oboe, clarinet, bassoon, violin, and cello) was premiered two weeks later in Bern and Basel by the musicians of Ensemble Proton.

Central to my work composing with respect to JI is a wish to create new harmonic music that is capable of modulation, rich in compositional possibilities at a range of speeds and affects, and most importantly, playable on acoustic instruments by live musicians, while featuring the distinctive sonority and pleasure afforded by rationally tuned sound combinations.